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DELIVERABLE

**D11.7 Report on the comparison of the explicit AHFM with
available test cases and on the implementation of the implicit
AHFM in OpenFOAM**

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The deliverable evaluates the various thermal turbulence models for simulating heat transfer in low-Prandtl fluids with Reynolds averaged Navier-Stokes approach. Three thermal turbulence models are of particular interest: the AHFM-NRG model, the Kays's correlation and the Manservigi's k_θ - ε_θ models. The performance of the three models is compared on two test cases: the infinite channel of Kawamura and the impinging jet case of Duponcheel and Bartosiewicz.

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1 Introduction

The aim of this deliverable is to evaluate various thermal turbulence models for simulating heat transfer in low-Prandtl fluids with Reynolds averaged Navier-Stokes (RANS) approach. The thermal turbulence models, analogous to momentum turbulence models which model the Reynolds stress terms $(\overline{u'_i u'_j})$ in the momentum equation, model the turbulent heat flux term $(\overline{u'_i T'})$ in the averaged heat equation (see for example [1]).

The performance of three models is of particular interest in the present context of low Prandtl fluids, such as liquid metals: the NRG version of the Algebraic Heat Flux Mode (AHFM-NRG) [2], the Kays's correlation [3] and Manservigi's k_θ - ε_θ model [4]. The aim is to use these models as is, without any particular tuning of parameters to the individual cases or special conditions.

The performance of thermal turbulence models of course depends also on the performance of momentum turbulence models. Some of the above mentioned thermal turbulence models were developed with specific momentum turbulence model in mind. For the sake of comparison, however, we disregard these connections and choose the most accurate momentum turbulence model for a particular case and use it with all the thermal turbulence models.

This deliverable presents the results obtained with these three models (in addition to others) on two cases for which data from a direct numerical simulation (DNS) is available. The comparison with the Kawamura's infinite channel dataset [5] is presented in Section 1.1. Since this case has two statistically homogeneous directions, the results can be presented on a single line, the wall normal direction.

The second case is the case of planar impinging jet of Duponcheel and Bartosiewicz [6]. The case is essentially 2 dimensional, however, it features three stagnation regions. The impingement of jet to the wall, and two reattachment zones of the recirculation bubbles. The results are presented in Section 1.2.

1.1 Kawamura case

This part of the deliverable presents the development of the Reynolds averaged Navier-Stokes (RANS) simulation of the Kawamura's direct numerical simulation (DNS) of the forced convection in the infinite channel flow.

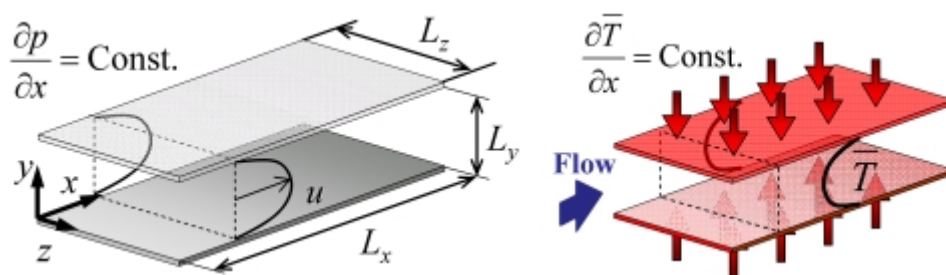


Figure 1: Momentum and thermal domain of Kawamura infinite channel [7].

The infinite channel, or Poiseuille flow is shown in Figure 1. The flow is trapped between two infinite planes. The characteristic length scale is the distance between the planes (wall-normal direction) and is equal to $2h$. In the statistical steady state, the average pressure drop throughout the domain is constant in the streamwise direction. The same is true for the average temperature rise in the streamwise direction.

The infinite channel flow is one of the simplest flows, since the statistically averaged result are essentially one-dimensional. That is, the time averaged results can be fully described by the distance

from the wall. Of course, the simulated channel is not infinite, but only a small portion of the domain is simulated, where the boundary conditions in the streamwise and span-wise direction are periodic. This contradicts with the above-mentioned constant average pressure drop and temperature rise through the domain. The linear pressure drop and linear temperature rise are taken care of, by transforming the Navier-Stokes equations. This is further described later with the numerical setup of the problem.

The DNS database itself can be found at [7] and a description of the equations can be found in [5]. A brief description is given here.

A channel flow problem can be described by the friction Reynolds number Re_τ , which is a Reynolds number calculated using the friction velocity u_τ as

$$Re_\tau = \frac{hu_\tau}{\nu}, \quad (1)$$

where ν is the kinematic viscosity of the fluid. The friction velocity, also known as shear velocity, is in turn calculated with the shear on the wall of the channel

$$u_\tau = \sqrt{\frac{\tau_w}{\rho}}. \quad (2)$$

The DNS data is available for various friction Reynolds numbers Re_τ and Prandtl numbers Pr . However, as we are interested in heat transfer of low-Prandtl fluid flows, we only selected a subset of available data, namely, two friction Reynolds numbers. Since the case is a forced convection flow, multiple passive scalars can be simulated along a single fluid field:

- $Re_\tau = 395$ with $Pr = 0.025, 0.71, 1, 2, 5, 7$ and 10 .
- $Re_\tau = 640$ with $Pr = 0.025$ and 0.71

1.2 Numerical setup

1.2.1 Description of the DNS setup

The details of the numerical setup of the DNS can be found in [5]. Here, only a brief description of the case is given. The coordinate system of the case is oriented so that the x axis corresponds to the streamwise direction. The y axis corresponds to the wall-normal direction. The orientation of the axes with the origin is shown in Figure 1.

The volume of the domain used in the DNS varies. In the case of lower friction Reynolds number, the size of the computational domain is $6.4h \times 2h \times 3.2h$, while in the case of higher friction Reynolds number the computational domain is $12.8h \times 2h \times 6.4h$.

The simulations are performed using the second order central finite difference method in case of $Re_\tau = 395$ for spatial discretization. For $Re_\tau = 640$ a fourth order central finite difference method was used in the streamwise and wall-normal direction, while a second order central finite difference method was used in the spanwise direction. For time discretization the Crank-Nicolson method is used for viscous terms in the wall-normal direction, while second-order Adams-Bashforth method is used for other terms.

With the lower friction Reynolds number, the resolution of the mesh in terms of wall units is 9.88 and 4.94 in the streamwise and spanwise directions, while it varies between 0.44 to 13.0 in the wall-normal direction. With the higher friction Reynolds number, the resolution of the mesh is 8.0 and 4.0 in the streamwise and spanwise directions, and between 0.15 and 8.02 in the wall normal direction.

The governing equations are the incompressible Navier-Stokes equations with added source terms to account for linear drops or increases in the streamwise direction.

$$\frac{\partial u_i^+}{\partial x_i^*} = 0, \quad (3)$$

$$\frac{\partial u_i^+}{\partial t^+} + u_j^+ \frac{\partial u_i^+}{\partial x_j^*} = -\frac{\partial p^+}{\partial x_i^*} + \frac{1}{\text{Re}_\tau} \frac{\partial^2 u_i^+}{\partial x_j^{*2}} + \delta_{i1}, \quad (4)$$

$$\frac{\partial \theta^+}{\partial t^+} + u_j^+ \frac{\partial \theta^+}{\partial x_j^*} = \frac{1}{\text{Re}_\tau \cdot \text{Pr}} \frac{\partial^2 \theta^+}{\partial x_j^{*2}} + \frac{u_1^+}{u_B}. \quad (5)$$

Here, the a^+ variables are quantities in wall units, normalized either with friction velocity u_τ (for velocity), h/u_τ (for time) or $q/(\rho u_\tau c_p)$ in case of temperature (q is the power per area of wall). u_B is the dimensionless bulk velocity in the channel. x_i^* are the distances normalize with channel halfwidth h . δ_{ij} is the Kronecker delta.

The boundary conditions for velocity field in the DNS are periodic in the streamwise and spanwise direction, while no slip boundary conditions are used at the wall. For temperature, the boundary conditions are periodic in streamwise and spanwise direction, while a constant temperature of 0 is imposed at the walls. However, due to how the dimensionless equations are derived, an average heat flux is also assumed at the walls and in addition to the momentum source, a volumetric heat source is also introduced. This allows for the temperature in the bulk flow to rise above the temperature of the walls. For details refer to Appendix A.

The following database files were used:

- Poi395_2nd_A.dat
- Poi395_2nd_A_uhf0025.dat
- Poi395_2nd_A_uhf071.dat
- Poi395_4th_D_uhf1.dat
- Poi395_4th_D_uhf2.dat
- Poi395_4th_D_uhf5.dat
- Poi395_4th_D_uhf7.dat
- Poi395_4th_D_uhf10.dat
- Poi640_4th_B.dat
- Poi640_4th_B_uhf0025.dat
- Poi640_4th_B_uhf071.dat

The bulk Reynolds numbers, calculated in the case of lower and higher friction Reynolds numbers in the DNS are 13967 (Poi395_2nd_A) and 24428 (Poi640_4th_B) respectively. In the case of Poi395_4th_D the bulk Reynold number calculated in the DNS is 14124, which is more than 1 % difference from the Poi395_2nd_A case. Nevertheless, only momentum field results from Poi395_2nd_A were used for comparison of RANS results.

1.2.2 Description of the RANS setup

The setup of the RANS simulation is similar to the setup of the DNS, however, there are subtle differences. The coordinate system in the RANS simulation is defined similarly as in the DNS. The domain has dimensions of $12h \times 2h \times 5h \cdot 10^{-3}$. Only one computational cell is employed in the spanwise (z) direction, while 50 cells are employed in the streamwise (x) direction resulting in a 2D simulation.

The equations solved by OpenFOAM are dimensional. The halfwidth of the channel is chosen as $h = 3.025$ cm. The viscosity was chosen to be $\mu = 1.74 \times 10^{-7}$ Pa s, the density $\rho = 1$ kg/m³ and the specific heat capacity $c_p = 1$ J/(kg K). The following equations are solved for the momentum field

$$\frac{\partial v_i}{\partial x_i} = 0, \quad (6)$$

$$\rho \frac{\partial v_i}{\partial t} + \rho v_j \frac{\partial v_i}{\partial x_j} = -\frac{\partial p_1}{\partial x_i} + \frac{\partial}{\partial x_j} \left(\mu \frac{\partial v_i}{\partial x_j} \right) + \frac{\Delta p}{h} \delta_{1i}, \quad (7)$$

where Δp is the linear pressure drop in the streamwise direction, needed to achieve a constant bulk velocity and p_1 is the pressure without this linear pressure drop. Note that $\Delta p > 0$. The Δp parameter is varied internally by OpenFOAM (through the use of `meanVelocityForce` source term) to achieve a steady state bulk velocity v_B . This velocity is an input parameter to the RANS simulation and is calculated from the bulk Reynolds numbers given in the DNS. The bulk Reynolds number is calculated as

$$\text{Re}_B = \frac{2h v_B}{\nu}. \quad (8)$$

The approximate values for friction and bulk velocities, calculated with the above-mentioned units, are given in Table 1.

Table 1: Approximate values for friction and bulk velocities calculated with units chosen for the RANS simulation.

Re_τ	Re_B	$u_\tau/(\text{mm s}^{-1})$	$v_B/(\text{cm s}^{-1})$
395	13967	2.27	4.02
640	24428	3.68	7.03

Since this implementation differs from the DNS and needs information from the DNS to perform the simulation, we also experimented with a source term that is similar to the one used in the DNS. The following equation is used in this case:

$$\rho \frac{\partial v_i}{\partial t} + \rho v_j \frac{\partial v_i}{\partial x_j} = -\frac{\partial p_1}{\partial x_i} + \frac{\partial}{\partial x_j} \left(\mu \frac{\partial v_i}{\partial x_j} \right) + \frac{\rho v^2}{h^3} \text{Re}_\tau^2 \delta_{1i}, \quad (9)$$

where the source term was written in the form which only depends on chosen units and parameters. This implementation is referred as *tau* in the figures in the Results section. The bulk velocity is not an input parameter in this implementation but is calculated from the velocity profile, while the friction velocity and the friction Reynolds number is better respected.

For both cases, the thermal equation is rewritten with the modified temperature

$$T_1 = T_0 - T + \frac{q}{\rho c_p h v_B} x_1, \quad (10)$$

where T_0 is a reference constant temperature and T is the dimensional temperature. Note that the temperature T_1 is inverted to regards to T and this follows the choice made by Kawamura. The temperature equation with this choice can be rewritten as

$$\frac{\partial T_1}{\partial t} + v_i \frac{\partial T_1}{\partial x_i} = \frac{\partial}{\partial x_i} \left(\alpha \frac{\partial T_1}{\partial x_i} \right) + \frac{T_d v}{h^2} \text{Re}_\tau \frac{v_1}{v_B}, \quad (11)$$

where only the chosen units and parameters are used. T_d is the ratio T_1/θ^+ , where θ^+ is dimensionless temperature in the DNS. The parameter T_d in our case is chosen to be 1 K, so that numerical values of T_1 (in units of K) correspond with numerical values of θ^+ (without units). The bulk velocity v_B is calculated as a volumetric average at each iteration, although it is assumed constant in the derivation of equations (in the time derivative term in the DNS, for example). Since our calculation is focused on steady state, this detail should not matter.

The boundary conditions for velocity field are periodic in the streamwise direction, empty (meaning the equation in this direction is not solved for) in the spanwise direction, while no slip boundary condition is imposed at the walls. For temperature fields, a constant heat flux is imposed at the walls:

$$n_j \frac{\partial T_1}{\partial x_j} = -\frac{T_d}{h} \text{Pr Re}_\tau, \quad (12)$$

where n_j is the wall normal vector. This boundary condition differs from the boundary condition in the DNS, where a constant temperature of 0 is imposed. However, this constant heat flux is implicit to the dimensionless equations. It was found that there is no significant difference in results between using either of the boundary conditions in the RANS simulation, nevertheless, the constant heat flux boundary condition converges to a solution somewhat sooner. The equivalence between prescribing constant temperature or constant heat flux at the wall also follows from theoretical considerations. The heat flux at the wall is defined by the volumetric heat source. If the heat flux at the wall and volumetric heat source mismatch, a steady state solution is impossible.

The mesh used in the RANS simulation, shown in Figure 2, consists of two blocks, with smaller cells at the walls and wider cells at the center of the channel. The blocks are symmetric across the centerline of the channel.

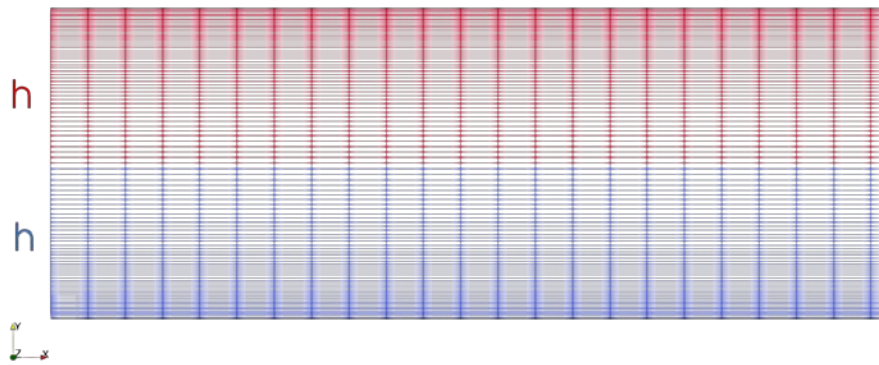


Figure 2: Mesh used for simulating the infinite channel flow in a RANS simulation.

The meshes we used range from 2500 to 10k cells. Generally, with 50 cells in the streamwise direction, although a study of the influence of number of cells in the streamwise direction was also performed (see Appendix D).

1.3 Results

In this section the comparison in predictions between different models is presented. First the comparison between the turbulence momentum models is shown. Based on the evaluation of the models the most suitable model was selected for simulations with thermal turbulence models. We are aware that different thermal turbulence models were developed with certain momentum turbulence models, however, our reasoning is that thermal turbulence model should not compensate for worse prediction of momentum field due to worse performance of the momentum turbulence model. Therefore, the momentum turbulence model with best prediction was selected to be used with all thermal turbulence models.

1.3.1 Comparison of turbulent momentum models

In this section the comparison between different momentum models, as well as the different momentum source terms are presented. The Launder-Sharma $k-\varepsilon$ model with *tau* source term, Launder-Sharma $k-\varepsilon$ model with *bulk* source term and $k-\omega$ SST model results are compared to results of the DNS. Since the *tau* implementation of the source term takes a bit longer to converge, but no significant advantage is gained over the *bulk* implementation, the $k-\omega$ SST model was only simulated with the *bulk* implementation.

The mesh convergence study can be found in Appendix D. The results presented in this section are obtained with the mesh with $N = 300$ for $Re_\tau = 395$ and mesh with $N = 400$ for $Re_\tau = 640$. The y^+ of the first mesh point for the lower Reynolds number is below 0.2, while for the higher Reynolds number it is slightly above 0.2. Figures in this section are only presented for $Re_\tau = 395$, while the results for $Re_\tau = 640$, which are similar and offer similar conclusions, are presented in Appendix C.

Figure 3 shows the comparison of the average streamwise velocity component between the three models for $Re_\tau = 395$. The τ implementation of the source term differs significantly from the other two models, as well as from the DNS results. This is the main reason the $k-\omega$ SST model was not attempted with the τ momentum source term.

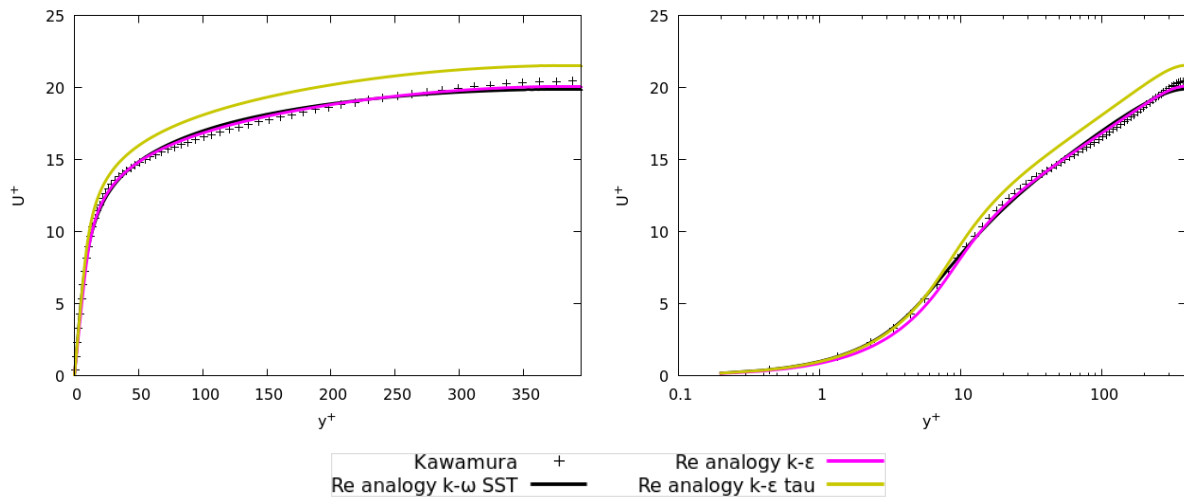


Figure 3: Comparison of average streamwise velocity component between Launder-Sharma $k-\epsilon$ model with bulk velocity source term, Launder-Sharma $k-\epsilon$ model with τ implementation of source term and $k-\omega$ SST model with bulk velocity source term for the case with $Re_\tau = 395$.

Figure 4 shows the turbulence kinetic energy and the dissipation of turbulence kinetic energy predicted by all three models. No model seems to predict better result than the other, although the Launder-Sharma $k-\epsilon$ model with the τ source term seems to be slightly better.

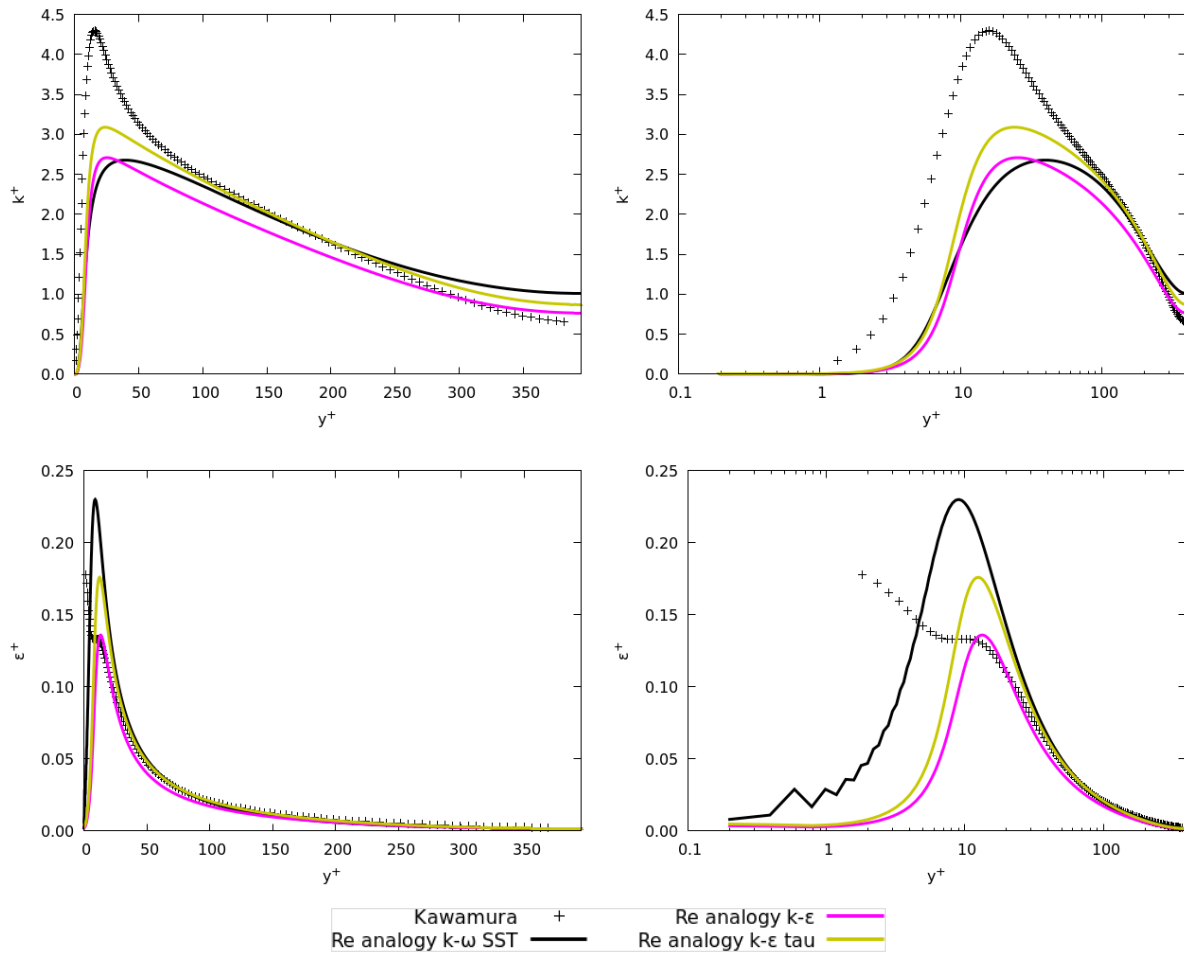


Figure 4: Comparison of turbulence kinetic energy (top) and the dissipation of turbulence kinetic energy (bottom) between Launder-Sharma $k-\varepsilon$ model with bulk velocity source term, Launder-Sharma $k-\varepsilon$ model with tau implementation of source term and $k-\omega$ SST model with bulk velocity source term for the case with $Re_\tau = 395$.

Figure 5 shows the eddy viscosity predicted by the three models. Here the Launder-Sharma $k-\varepsilon$ model with the *bulk* source term seems to be better, while the $k-\omega$ SST model prediction is the worst.

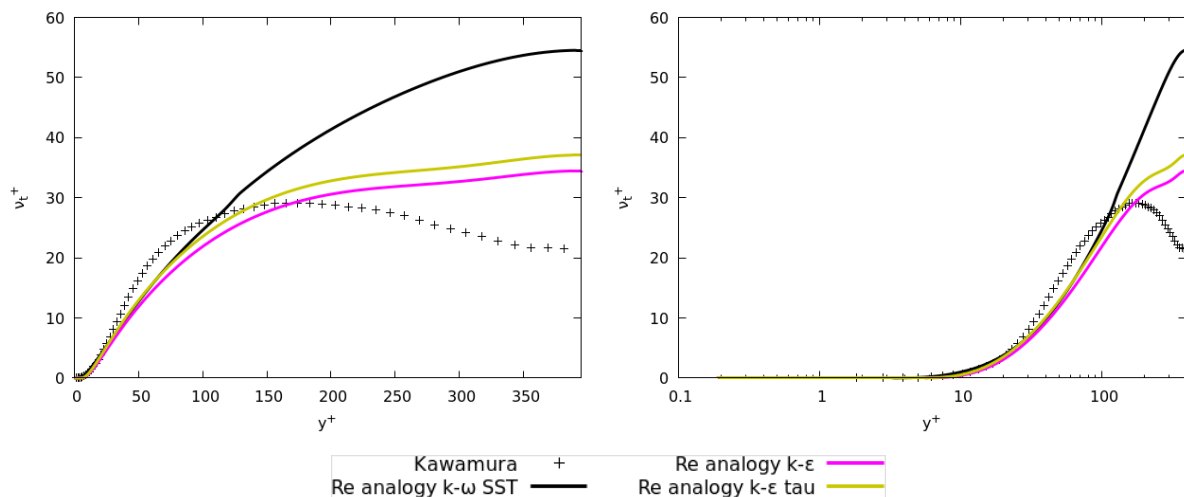


Figure 5: Comparison of turbulence eddy viscosity between Launder-Sharma $k-\varepsilon$ model with bulk velocity source term, Launder-Sharma $k-\varepsilon$ model with tau implementation of source term and $k-\omega$ SST model with bulk velocity source term for the case with $Re_\tau = 395$.

Table 2 presents the calculated friction Reynolds number and the bulk Reynolds number. While the τ source term captures the friction Reynolds number of the DNS by design, it overpredicts the bulk Reynolds number. From the two *bulk* implementations, which capture the bulk Reynolds number perfectly, the $k-\omega$ SST model is much closer with the prediction of the friction Reynolds number. Based on the presented results, the $k-\omega$ SST model was selected as the base model to be used with the thermal turbulence models.

Table 2: Calculated friction and bulk Reynolds numbers for fields with different models for the case with $Re_\tau = 395$.

Model	Re_τ	Re_b
DNS	395	13967
Launder-Sharma $k-\varepsilon \tau$	394.6	15000.6
Launder-Sharma $k-\varepsilon$	370.4	13967.0
$k-\omega$ SST	398.4	13967.0

Similar considerations can be made for the higher friction Reynolds number 640. The details can be found in Appendix B, here, only the Reynolds numbers are shown in Table 3. The results lead to a similar conclusion and the $k-\omega$ SST model was chosen to be used with the thermal turbulence models.

Table 3: Calculated friction and bulk Reynolds numbers for fields with different models for the case with $Re_\tau = 640$.

Model	Re_τ	Re_b
DNS	640	24428
Launder-Sharma $k-\varepsilon \tau$	639.5	25721.7
Launder-Sharma $k-\varepsilon$	608.5	24428.0
$k-\omega$ SST	646.3	24428.0

1.3.2 Comparison of thermal turbulence models

In this section the results obtained for thermal and related field with different thermal turbulence models are presented. All the models were combined with the $k-\omega$ SST momentum turbulence model, as concluded in the previous section.

The following thermal turbulence models were applied:

- Reynolds analogy model with turbulent Prandtl number $Pr_t = 0.85$,
- Kays correlation with the model $Pr_t = 0.7/Pe_t + 0.85$ [3],
- Manservigi's $k_\theta-\varepsilon_\theta$ [4],
- Manservigi's $k_\theta-\omega_\theta$ [8],
- AHFM model as suggested by Kenjeres et al. [9],
- AHFM-NRG model [2], and
- Kays correlation with the model $Pr_t = 2/Pe_t + 0.85$.

Only some of the results obtained with these models are presented in this section. Additional figures for the missing Prandtl numbers and figures regarding $Re_\tau = 640$, supporting conclusions drawn in this section can be found in Appendix C.

An example of temperatures obtained with these models is presented in Figure 6 for Prandtl numbers 0.025, 0.71 and 2. Nevertheless, a more concise way to present results is through an integral measure of discrepancy, like

$$\varepsilon = \frac{1}{Re_\tau} \int_0^{Re_\tau} \left| \frac{T^+ - T_{DNS}^+}{T_{DNS}^+} \right| dy^+.$$

Results of the model and DNS are interpolated with third degree splines to common points along the channel line, where the fraction is calculated. Simpson's method is used to evaluate the integral.

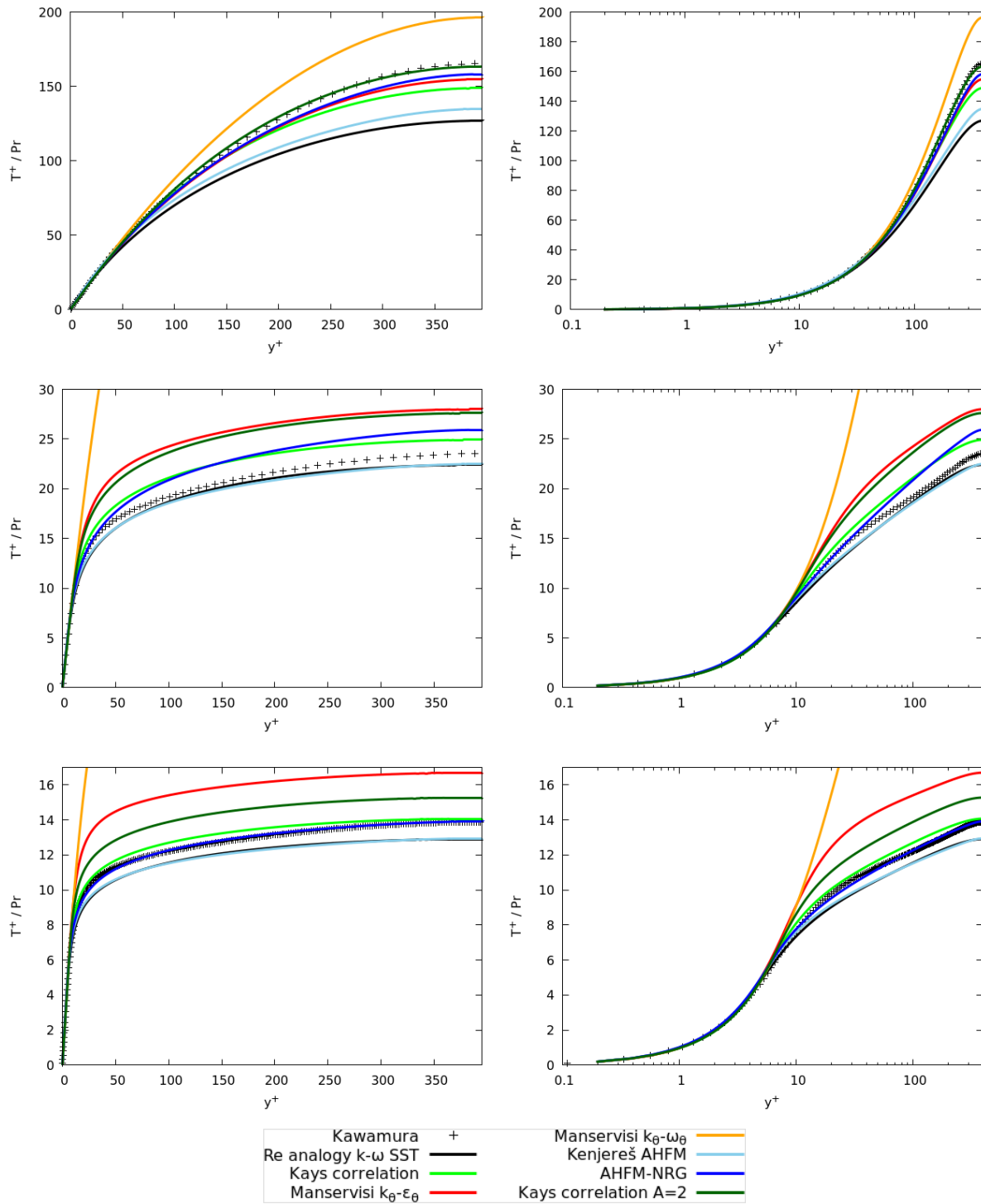


Figure 6: Temperature distribution predicted by different thermal turbulence models for $Pr = 0.025$ (top), $Pr = 0.71$ (middle) and $Pr = 2$ (bottom).

The above norm is shown for all Prandtl numbers and both friction Reynolds numbers in Figure 7. At the lowest Prandtl number all the models are behaving adequately, with a clear bias towards the models developed for low Prandtl numbers. The low Prandtl version of Kays correlation seems to match the best with the DNS results. The Manservisi's models seem to perform the worst, overall. Even worse than the Reynolds analogy. The models were developed with the $k-\varepsilon$ momentum models and the parameters were tuned with that model in mind, and it might therefore be expected that performance with the $k-\omega$ SST model suffers. The models performing best over all the Prandtl numbers are the general Kays correlation and the AHFM-NRG. Of these two, the authors of this report would

choose the Kays correlation, since the AHFM-NRG model needs some minor case specific parameter inputs (the products $Re Pr$ and $Ra Pr$) and is very sensitive to the initial conditions. The results could not be obtained by starting from zero initial conditions, which is the case with other models. Nevertheless, the zero initial condition was not used with other models either (although it could have been) in order to speed up the convergence of the results.

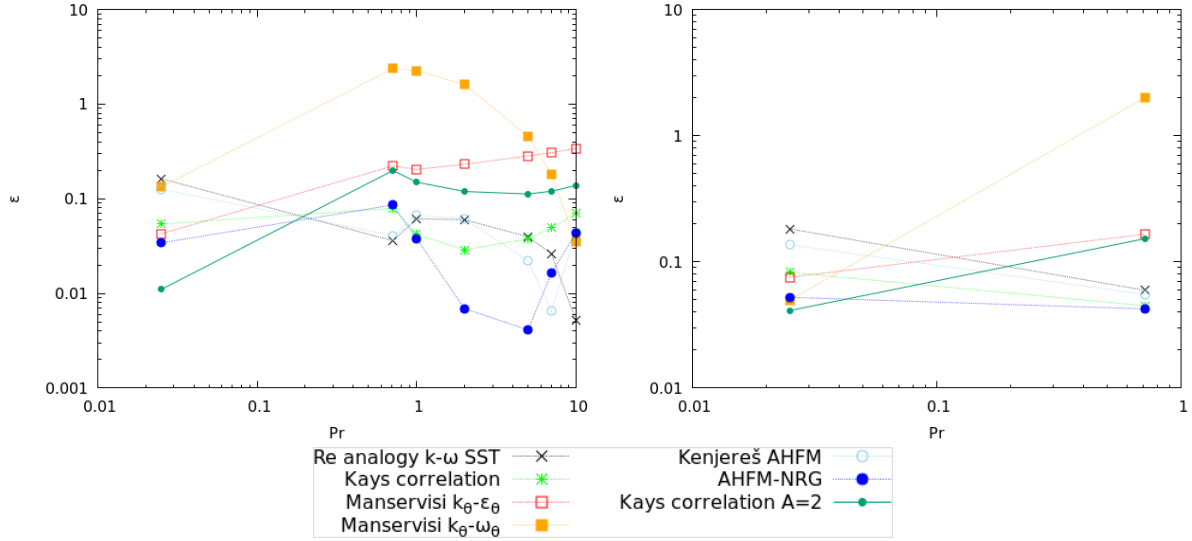


Figure 7: Discrepancy between temperature results obtained with models and DNS results for $Re_\tau = 395$ (left) and $Re_\tau = 640$ (right). The lines are strictly to guide the eye.

The performed simulations show that considering temperature distribution over all models (excluding the Manservisi's $k_\theta-\omega_\theta$), the maximal absolute difference of T^+/Pr compared to the DNS is contained within around 5 units for higher Prandtl numbers irrespective of the two Reynolds numbers used. At higher Prandtl numbers this difference is even smaller (down to 2.3 at $Pr = 10$). Nevertheless, at Prandtl number 0.025, this difference to DNS results is around 30 and 40 for the lower and higher Reynolds number, respectively.

As a general observation, since the diffusion term in the thermal equation is of the form

$$\frac{\partial}{\partial y} \left((\alpha + \alpha_t) \frac{\partial T}{\partial y} \right),$$

it seems the modelling of turbulent thermal diffusion does not play a large role at low Prandtl numbers, since it is comparable to the molecular thermal diffusion (see Figure 8). The temperature gradient is nevertheless significant. On the other hand, the only large thermal gradient at high Prandtl numbers is close to the wall. Away from the wall, the temperature profile is flat. The ratio of turbulent and molecular thermal diffusivities is nevertheless noteworthy in the advantage of turbulent thermal diffusivity already close at the wall.

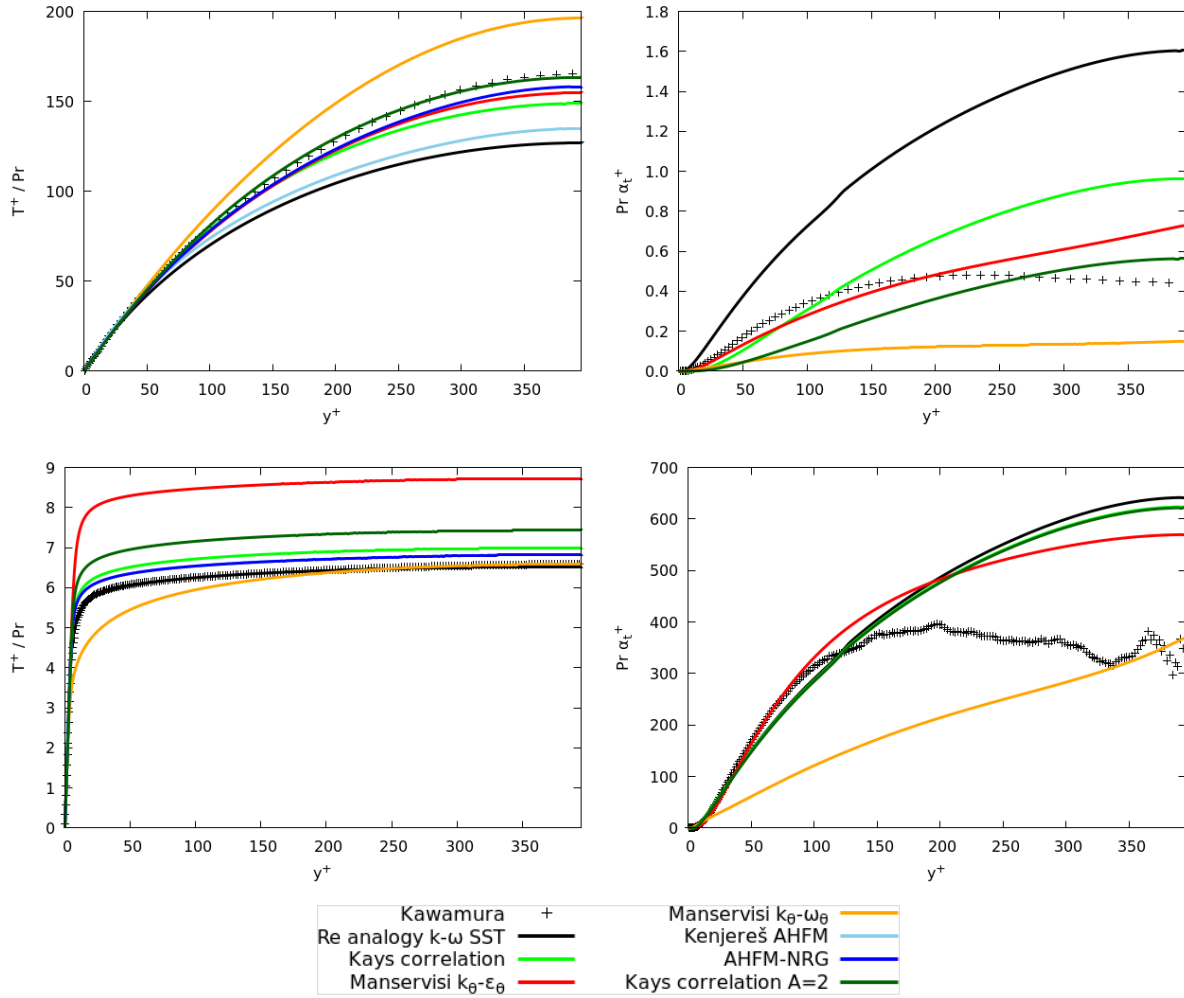


Figure 8: Temperature (left column) and ratio of turbulent and molecular thermal diffusion (right column) for Prandtl numbers 0.025 (top) and 10 (bottom) at $Re_\tau = 395$.

1.4 Conclusions

In this section of the deliverable, the six thermal turbulent models were applied to the Kawamura channel flow case. Two friction Reynolds numbers (395 and 640) and different Prandtl numbers were simulated. The $k-\omega$ SST momentum turbulence model was selected over the Launder-Sharma $k-\epsilon$ model, to be used with all thermal turbulence models since it captured the friction and bulk Reynolds numbers of the DNS most accurately. This does seem to put the thermal momentum models that were developed with a $k-\epsilon$ type momentum model (such as the two Manservisi's models) at a disadvantage. Nevertheless, at the lowest Prandtl number, all the models performed better as the Reynolds analogy.

The most accurate thermal turbulence models for this forced convection case are the AHFM-NRG model and the Kays correlation. Of these two, the Kays correlation does not require any parameter tuning, is fast and can be used with arbitrary initial conditions. The AHFM-NRG model requires some parameter inputs (Re Pr), and is very sensitive to initial conditions.

2 Impinging jet

In this section the comparison of temperature prediction of different thermal turbulence models is compared in a case of the impinging jet of Duponcheel and Bartosiewicz [6]. Although the DNS reference database contains data for two Reynolds numbers (4000 and 5700), we present only results for the higher Reynolds number. Comparison of some thermal turbulence models for the lower Reynolds number was performed by De Santis et al. [10].

In this case three momentum turbulence models were evaluated: the Launder-Sharma $k-\varepsilon$, the $k-\omega$ SST, and the v2-f model, as the latter was developed explicitly for impinging jets in mind. Of the thermal turbulence models we evaluated the performance of Reynolds analogy with $Pr_t = 0.85$, Kays's correlation, Manservigi's $k_\theta-\varepsilon_\theta$ and AHFM-NRG models. No tuning of parameters was performed.

The details of the DNS setup will not be repeated here since it is readily available in the literature. The set-up is purely forced convection and the temperature is treated as a passive scalar. The turbulent inlet is simulated in a parallel, but separate, infinite channel domain with periodic boundary conditions. The resulting channel velocity profile of some plane is then imposed as the inlet boundary condition in the middle of the top plate (see Figure 9). The dimensionless temperature of the fluid at the inlet is at constant 1, while at the walls the constant dimensionless temperature is set to 0. At the outflow, constant pressure is imposed.

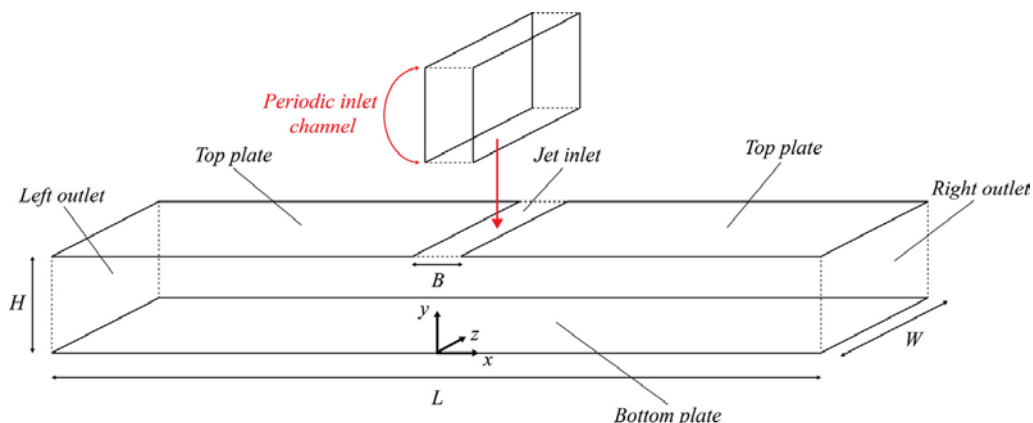


Figure 9: Geometry of impinging jet database [10].

2.1 Numerical setup

Geometry of the RANS setup is shown in Figure 9. It matches the geometry of the DNS, except it is a little shorter. The inlet channel width is B . The bottom wall is of length $L = 40B$, while the height of the domain is $H = 2B$ and the thickness of the domain is $W = 3.14B$. The bottom wall in the DNS is of length $L = 80B$.

The inflow boundary condition in the RANS simulation is generated with a channel flow simulation, similar to the simulations presented in previous sections. The velocity (u_i) and turbulent fields (k , ε or ω depending on the model, and ν_t) are then imposed as an inlet boundary condition. The comparison of velocity profile and the profile of turbulence kinetic energy is shown in Figure 10. The quantities are normalized with the friction velocity. In the DNS the bulk Reynolds number is 5700 and the friction Reynolds number is equal to 181.5. The temperature at the inflow is constant 1 and the walls have a constant temperature of 0, as in the DNS.

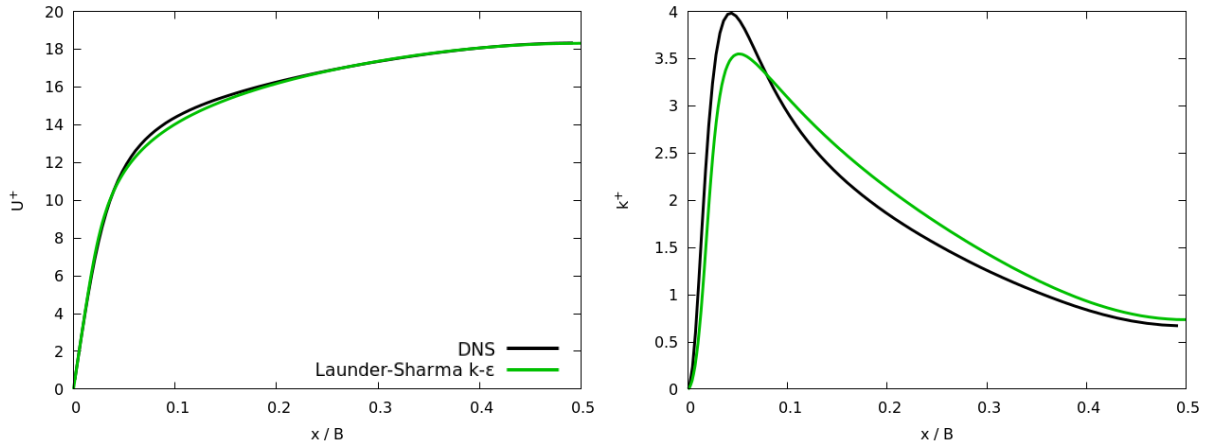


Figure 10: Inlet velocity profile, normalized with friction velocity (left) and inlet turbulence kinetic energy normalized with square of friction velocity. (right)

Mesh sensitivity study was performed in [10] and was not performed anew here. The mesh had 80 cells in the x direction in the inflow region and 472 in the x direction of each channel. In the y direction there were 144 cells. The y^+ of the first cell is below 1.7 everywhere in the domain.

2.2 Results

In this section, RANS results are compared to the DNS results. First the momentum fields are presented. Figure 11 shows the skin friction along the bottom wall. The peak below $x/B \sim 1$ is captured well by all the three models, while the second peak around $x/B \sim 5$ is not well reproduced. This can be observed also in the plot of difference compared to the DNS results on the left side of Figure 11. Since the second peak is not well captured the difference is quite large in the location of second peak.

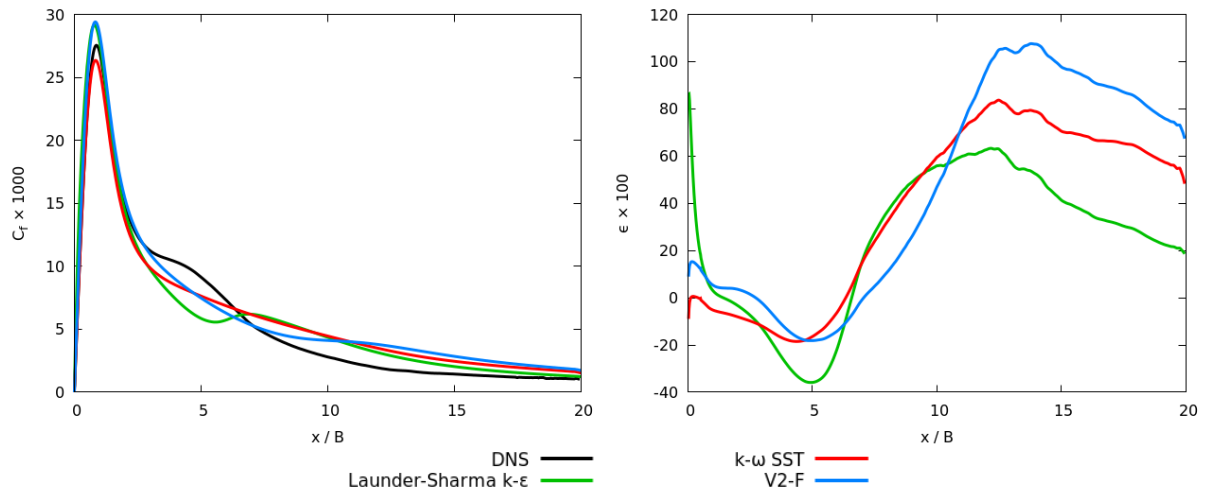


Figure 11: Skin friction coefficient along the bottom wall (left) and discrepancy of wall shear stress between RANS simulation and the reference data (right).

Figure 12 presents the streamwise velocity component comparison with DNS. The streamwise component is predicted quite well by all three momentum turbulence models. The prediction matches the DNS well around the impingement area. The backflow towards the jet near the top wall is slightly overpredicted. The prediction becomes worse toward the outflow of the domain. The local discrepancy is drawn at the bottom of Figure 12 and we can see the prediction of the streamwise component by the $k-\omega$ SST model is consistently worse than the other two models. Surprisingly, the most accurate prediction of the streamwise velocity component is given by the Launder-Sharma $k-\epsilon$ model.

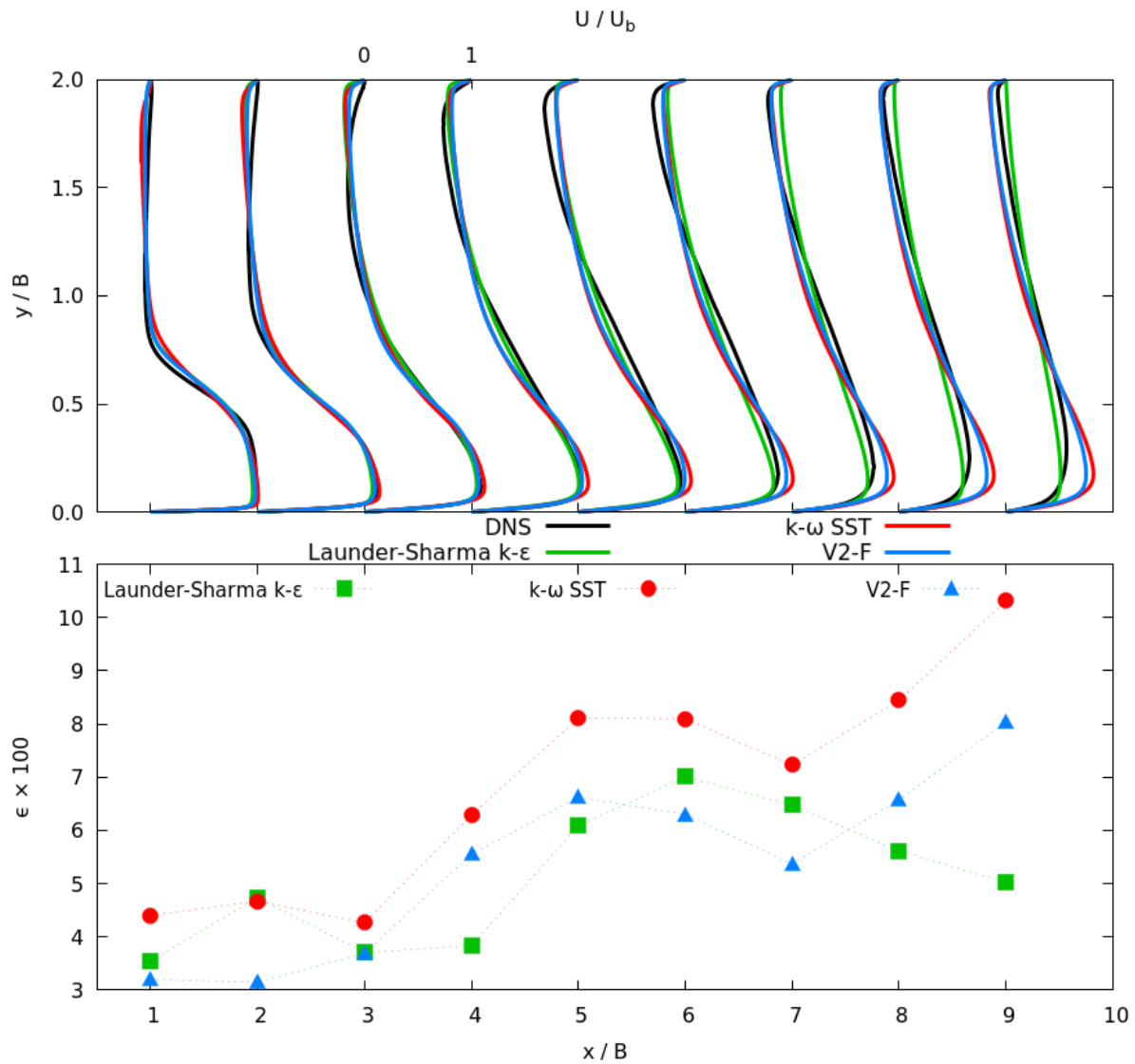


Figure 12: Streamwise velocity component (top) and discrepancy to reference results (bottom).

Figure 13 shows the turbulence kinetic energy compared to the DNS results. The predictions do not match well the results of the DNS close to the impinging area. The sharp peaks of the DNS are not well matched. Downstream of the jet area the prediction becomes better. The discrepancies are, however, an order of magnitude smaller than that for the streamwise velocity component. Nevertheless, here it can also be observed, that the $k-\omega$ SST model is performing the worst, while the Launder-Sharma $k-\epsilon$ model is performing better.

In Figure 14 the $u'v'$ component of the Reynolds stress is shown. The positive peak (near the bottom wall) and the negative peak (in the middle of the domain) are captured reasonably well with all models. The discrepancies are approximately the same for all the models as well.

In general, the result of the comparison is similar to the comparison by De Santis et al. [10] obtained for the Reynolds number 4000. Based on these results we decided to present results for thermal fields for Launder-Sharma $k-\epsilon$, as well as for v2-f models.

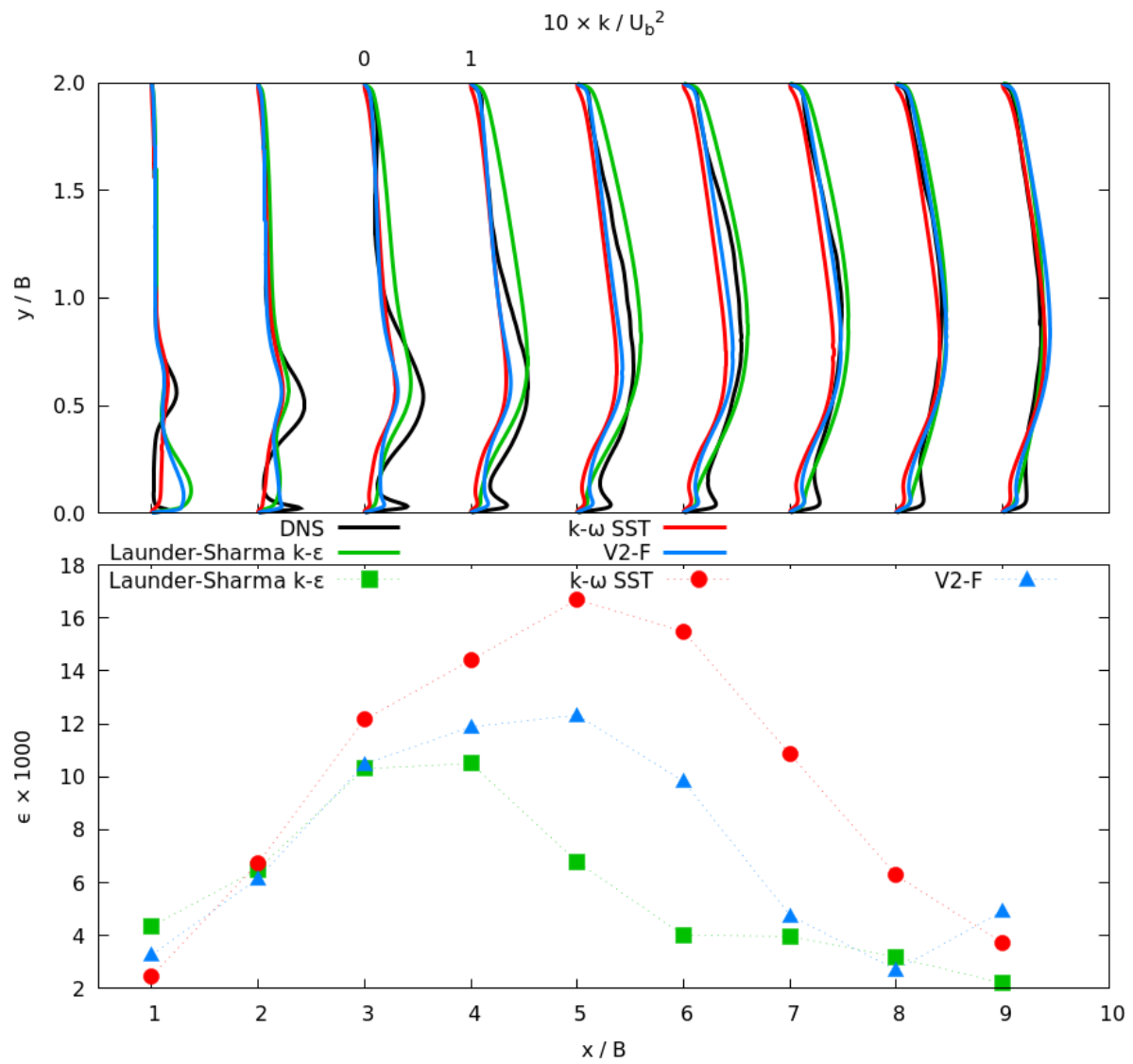


Figure 13: Turbulence kinetic energy (top) and the discrepancy of RANS results to reference database (bottom).

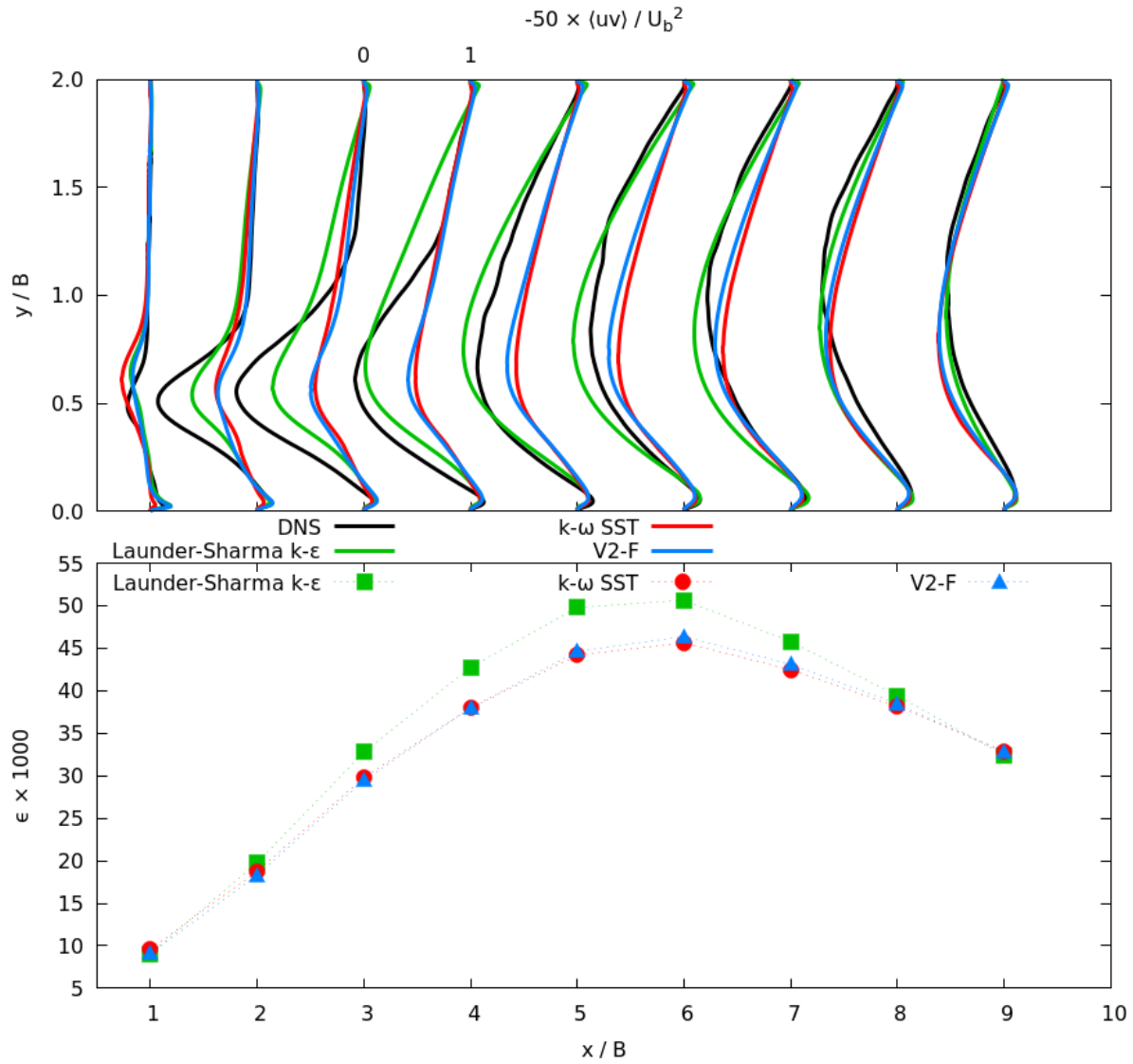


Figure 14: Reynolds shear stress (top) and the discrepancy of RANS results to reference database (bottom).

In Figure 15 the predictions of the Nusselt number of four thermal turbulence models with the two momentum turbulence models are compared to the results of the DNS. The difference to the DNS is also shown. The Reynolds analogy, as well as the Kays's correlation and the AHFM-NRG with the Launder-Sharma $k-\epsilon$ exhibit a spike close to the impinging site. This is similar to the skin friction coefficient shown in Figure 11.

With the AHFM-NRG model the following algebraic equation

$$\overline{u_i t} = -C_{t_0} \frac{k}{\epsilon} \left(C_{t_1} \overline{u_i u_j} \frac{\partial T}{\partial x_j} + C_{t_2} \overline{u_j t} \frac{\partial U_i}{\partial x_j} + C_{t_3} \beta g_i \overline{t^2} \right) + C_{t_4} a_{ij} \overline{u_j t}$$

is solved for $\overline{u_i t}$. a_{ij} is the Reynolds stress anisotropy tensor $a_{ij} = u_i u_j / k - 2\delta_{ij}/3$ and β is the thermal expansion coefficient. C_{t_0} , C_{t_1} , C_{t_3} and C_{t_4} are the model coefficients. The value of C_{t_0} is 0.2, C_{t_1} is set through the expression $C_{t_1} = 0.053 \ln(Re Pr) - 0.27$ for $Re Pr > 180$. For the Pr value of 0.01, this product is well below (57) the limit and the coefficient C_{t_1} becomes negative. For this reason, we limited C_{t_1} from below to 0. C_{t_2} is set to 0.6. Since this is a forced convection case, the coefficient C_{t_3} is irrelevant. Coefficient C_{t_4} is, similar as in [2], set to 0. We can note here that a non-zero value for C_{t_4} leads to inversion matrix for the above system to become singular. The values of the coefficients would need to be tuned so that the system would also be homogeneous at locations, where the

inversion matrix is singular, to allow for solutions. However, since in the literature C_{t_4} is anyway zero and the tuning of coefficients is not the focus of this task, no such attempt was made.

Figure 15 shows the Nusselt number for Prandtl number 1 along the bottom wall. Both the Manservisi's model and AHFM-NRG show better results as Reynolds analogy and Kays's correlation. Nevertheless, the scale of the relative difference is in few 10s of percent. The peak around 5 is not captured by any model. The results obtained with the v2-f model are also shown with faintly colors. If the thermal turbulence models are ranked by how close to the DNS the prediction is, the ranking changes when the turbulence momentum model is changed.

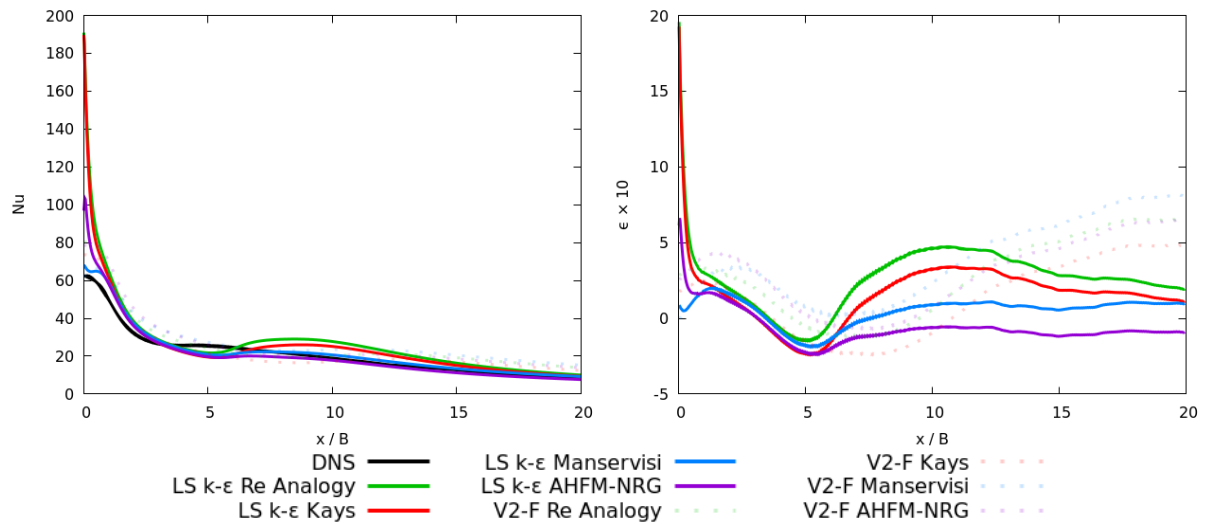


Figure 15: Nusselt number along the bottom wall (left) and the discrepancy from reference data (right) for passive scalar with $Pr = 1$.

The temperature profile obtained at Prandtl number 1 is shown in Figure 16. The temperature in the region close to the impingement is quite well predicted by all models. However, the predictions are off in the recirculation zone (that is, at the top wall). Also, downstream the thermal boundary region is underpredicted and the models predict a constant temperature closer to the wall, compared to the DNS.

From the difference to the DNS graph, it can be observed that the most well performing model is Reynolds analogy, which better captures the prediction in the recirculation zone. The worst performing is the Manservisi's model, followed by Kays's correlation. The discrepancy to the DNS temperature results has a peak at around $x \sim 2B$ for all the models, except the AHFM-NRG with v2-f momentum turbulence model. In common to other models the temperature discrepancy increases towards the outflow.

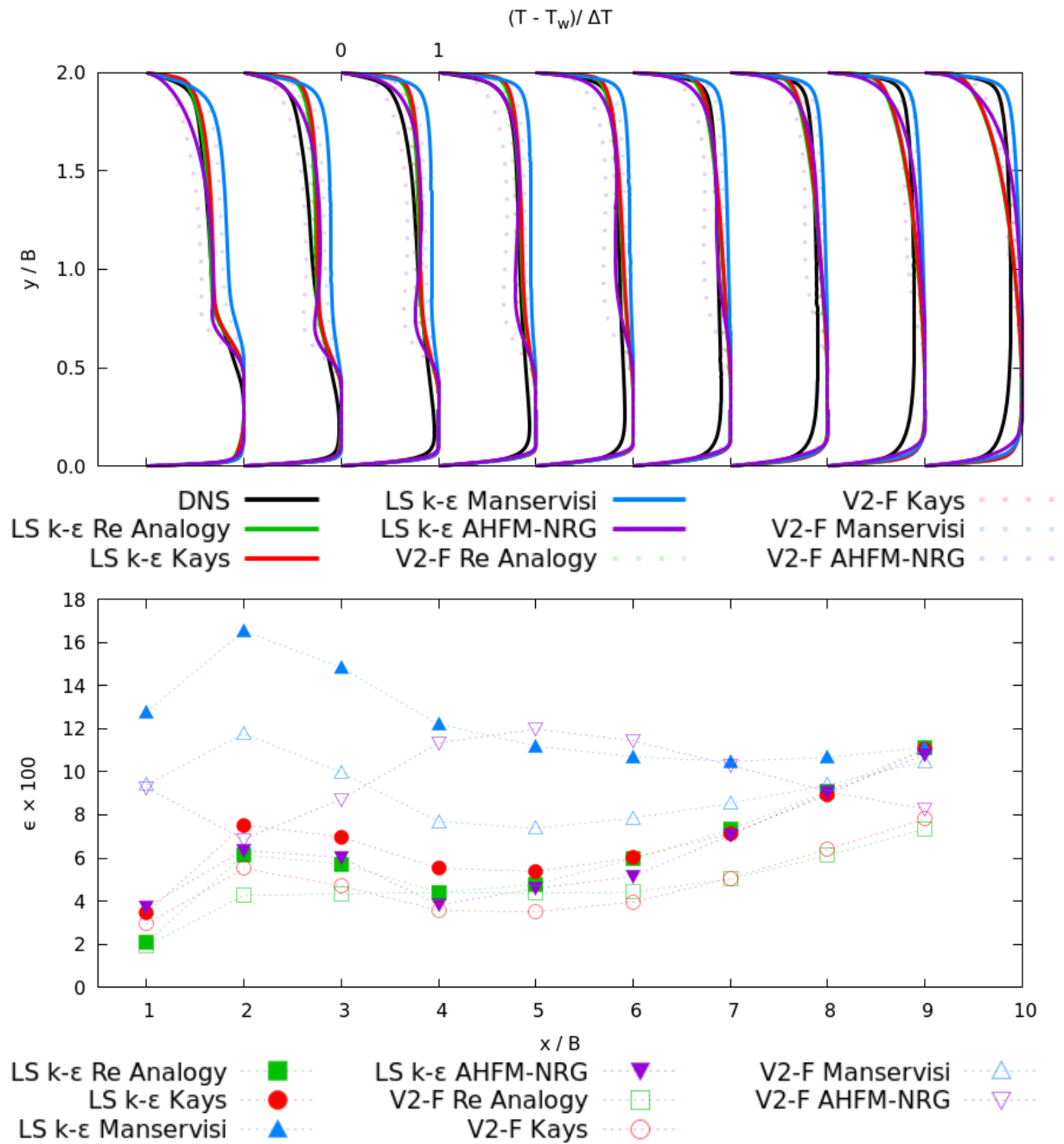


Figure 16: Temperature field (top) and discrepancy with reference data results (bottom) for $Pr = 1$.

Figure 17 shows the streamlines in one half of the domain. The color of the streamlines denotes the temperature of the fluid with Prandtl number 1 as predicted by Reynolds analogy. The considerable change in the reattachment length can be observed when considering the turbulence momentum models.

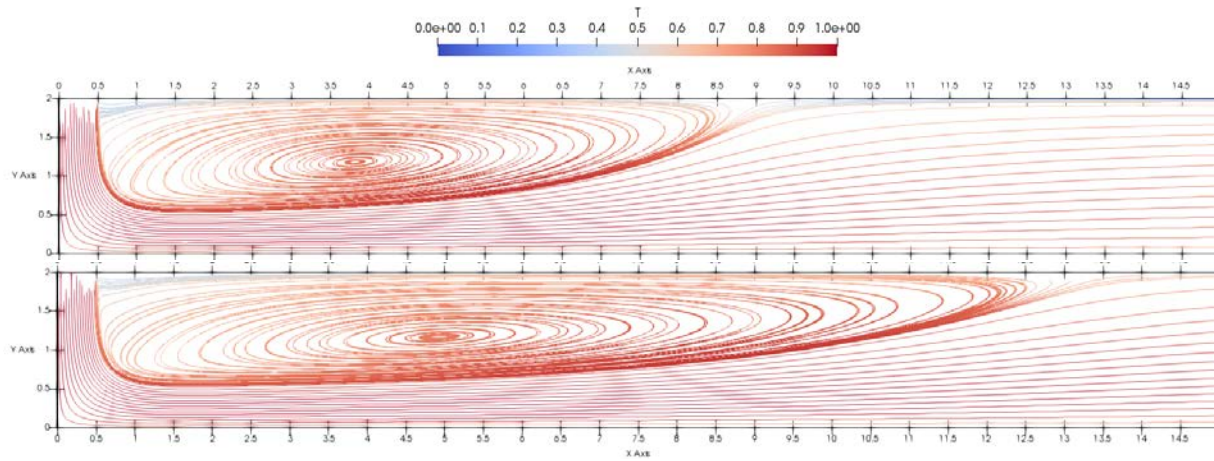


Figure 17: Shape of the recirculation zone with Launder-Sharma $k-\epsilon$ model (top) and $v2-f$ model (bottom) with temperature field for fluid with $Pr = 1$ obtained with Reynolds analogy.

Figure 18 shows the Nusselt number along the bottom wall for Prandtl number 0.1. The predictions of all the models are slightly better, but the order of models based on the difference to DNS is the same. The AHFM-NRG underpredicts the value of the peak at impingement, however, the peak at around $x/B \sim 7$ overpredicted by other models. The latter seems to be the biggest discrepancy compared to DNS.

In general, the results for $v2-f$ model are similar to those with Launder-Sharma $k-\epsilon$ model. Since with the $v2-f$ model, the second peak in the Nusselt number is lower, the peak in the difference to the DNS is not so apparent.

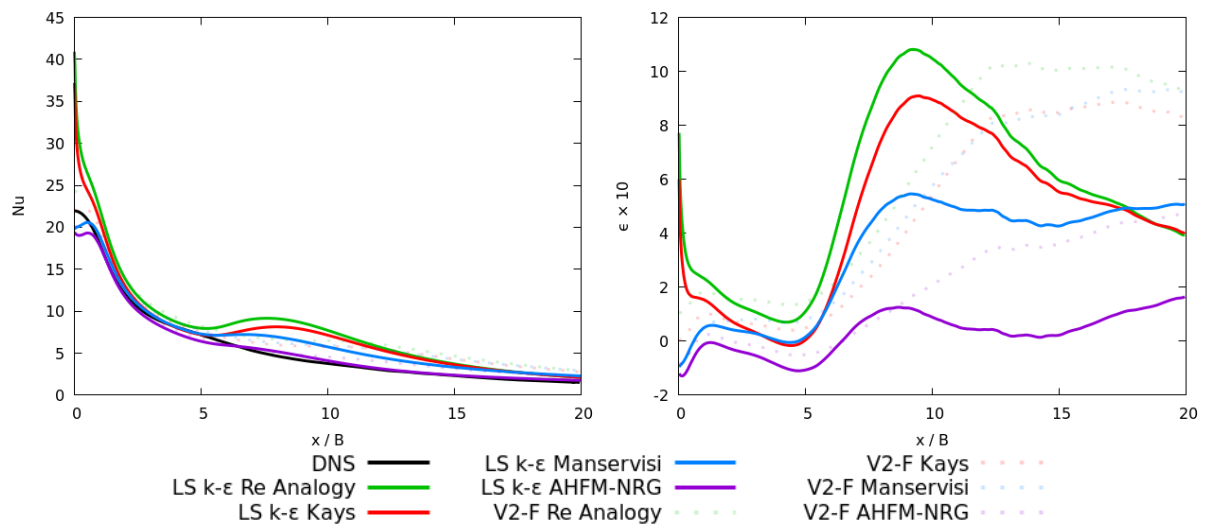


Figure 18: Nusselt number along the bottom wall (left) and the discrepancy from reference data (right) for passive scalar with $Pr = 0.1$.

Figure 19 shows the profile of temperature and the difference from the DNS for Prandtl number 0.1. Like in the case for $Pr = 1$ the temperature is captured well at the impingement, however, it is quite overpredicted in the recirculation zone, as well as downstream of the inflow. This might imply that the recirculation zone is too poorly predicted by the momentum turbulence Launder-Sharma $k-\epsilon$ model. The relative differences to the DNS are larger with the Launder-Sharma $k-\epsilon$ model and prediction with

the v2-f model seems somewhat better. The discrepancy from DNS temperature profile grows towards the outflow for all models.

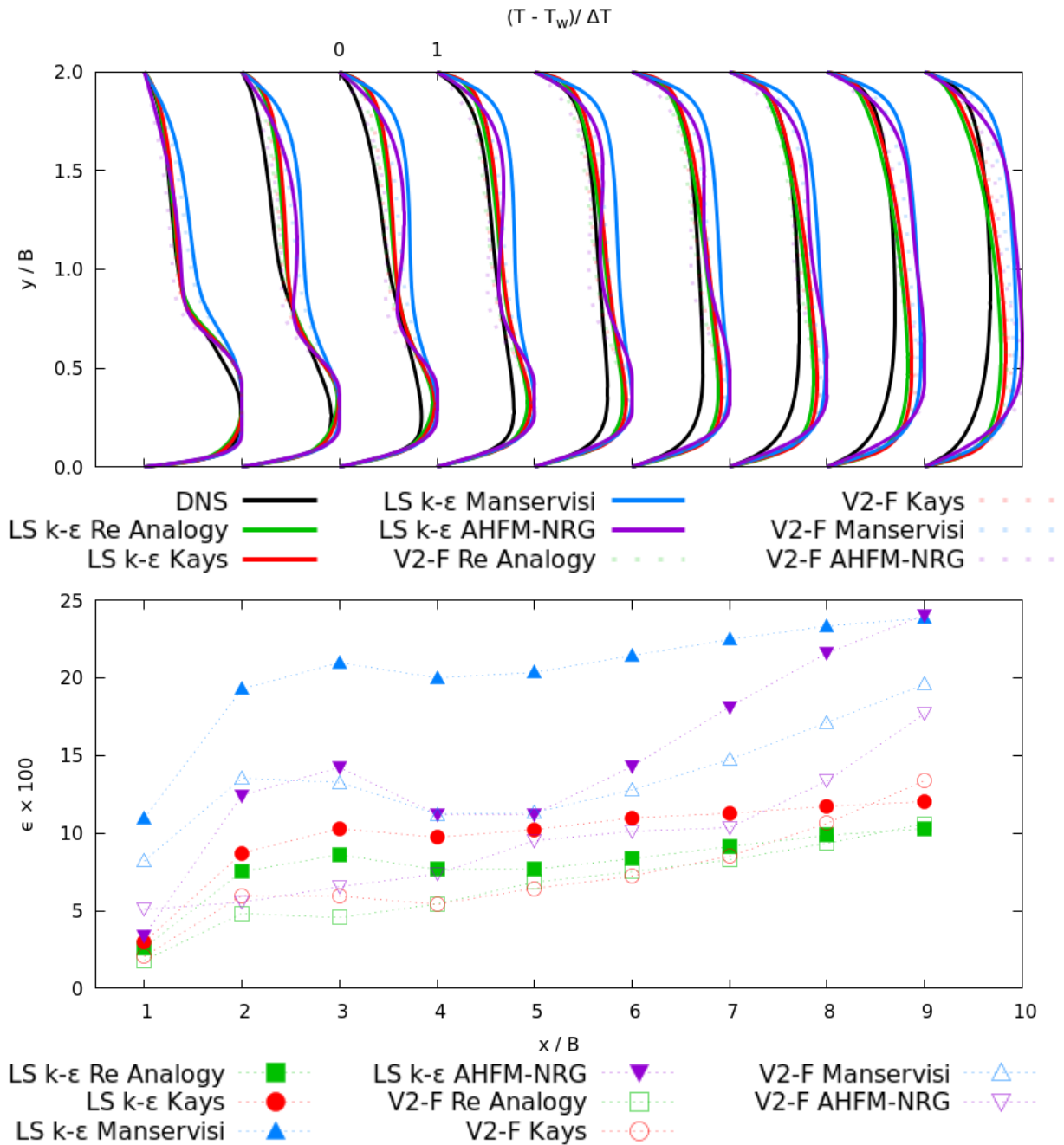


Figure 19: Temperature field (top) and discrepancy with reference data results (bottom) for $Pr = 0.1$.

Figure 20 shows the prediction of Nusselt number along the bottom wall for Prandtl number 0.01. The DNS predicts a single peak at the impingement. The models, however, predict a second peak at $x \sim 7B$ and thus the difference to the DNS results is quite large. Although there are small absolute differences between the models downstream of this region, the relative difference is quite large and maybe therefore not the most representative. The ranking of the thermal turbulence models according to the difference of temperature from DNS result with the v2-f momentum turbulence model at this Prandtl number is the same as with Launder-Sharma $k-\epsilon$ model.

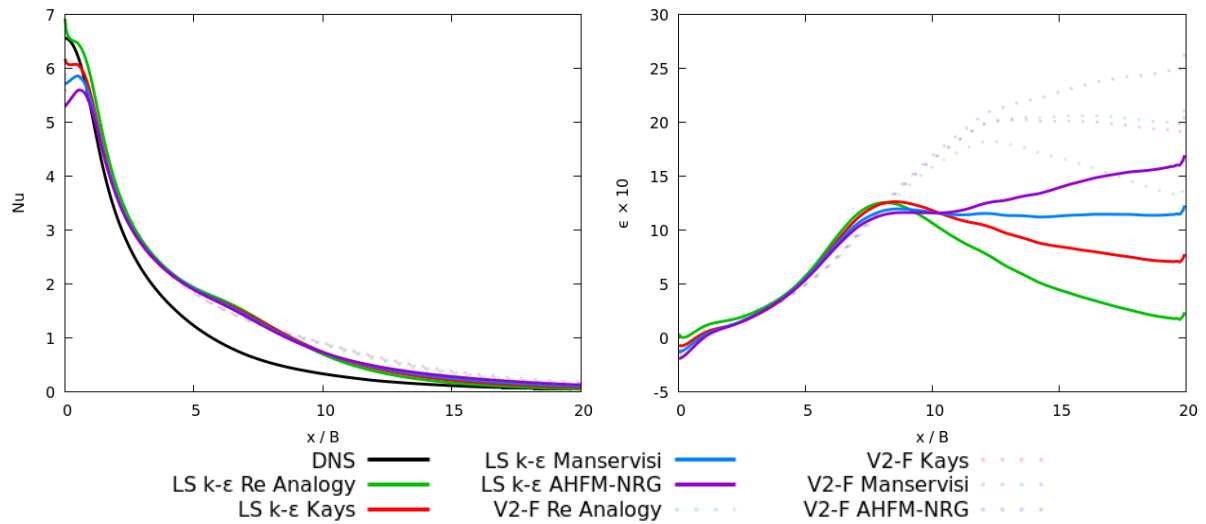


Figure 20: Nusselt number along the bottom wall (left) and the discrepancy from reference data (right) for passive scalar with $Pr = 0.01$.

Figure 21 shows the temperature profile through the domain for Prandtl number 0.01. Here the temperature in the recirculation zone seems to be better predicted. Surprisingly, the best prediction comes from the Reynolds analogy, as before.

The shape of the discrepancy from DNS prediction of temperature profile strongly depends on the momentum turbulence model here. The models with Launder-Sharma $k-\epsilon$ momentum turbulence model exhibit a drop in the discrepancy towards the outflow, while with the v2-f momentum turbulence model, the discrepancy to DNS results rises. This is consistent with the difference between streamwise velocity component prediction for the two momentum turbulence models.

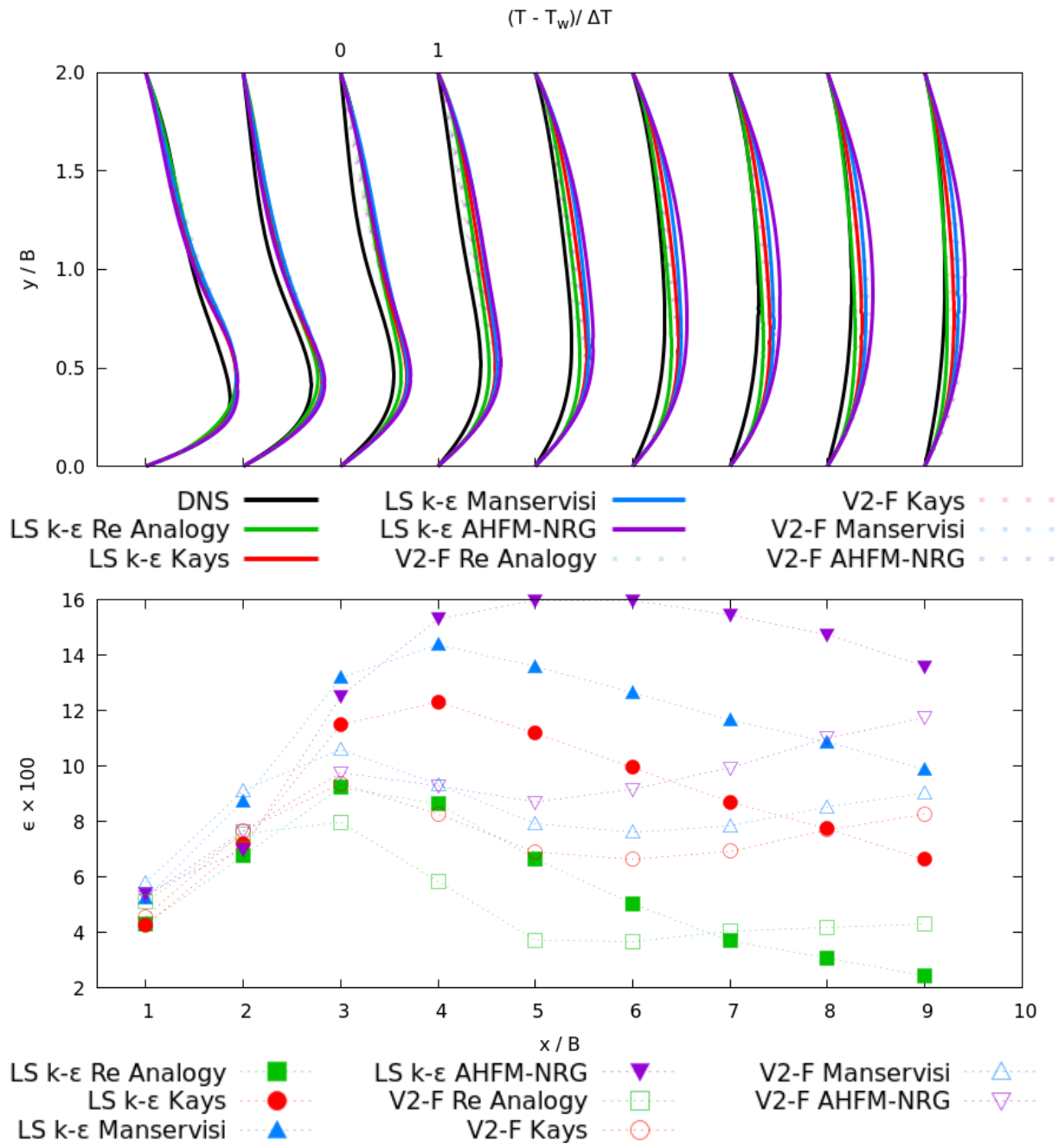


Figure 21: Temperature field (top) and discrepancy with reference data results (bottom) for $Pr = 0.01$.

2.3 Conclusions

We performed RANS simulations of different momentum turbulence models in combination with different thermal turbulence models and compared it to the results of DNS of a planar impinging jet, performed by Duponcheel and Bartosiewicz [6]. The predictions of the models seem to agree with DNS results within 25% for the temperature profile, while they can be by a factor different for the Nusselt number. Nevertheless, the Reynolds number of 5700 is quite low for turbulence modelling approaches. Due to this, applying the conclusions of these simulations to the fully turbulent flows might be difficult to justify.

Surprisingly, the model with overall best prediction of the temperature profile for this case seems to be the Reynolds analogy. It is also the best prediction in case of the lowest Prandtl number. This might be the consequence of the focus of parameter tuning of different models. For example, the Manservisi's model was introduced on the infinite channel flow, where the performance of the model

is better than of Reynolds analogy. It might be an indication that the model should be returned to a more general set of cases, however, this might also mean a loss of accuracy in the infinite channel flow case. Similar considerations could be made for other models.

From the results it also seems, the accuracy of the models is divided between prediction of the Nusselt number (i.e. the gradient of temperature) or the temperature profile. The order in which the models can be ranked according to how close to DNS results the prediction is for the temperature profile, is in reverse compared to the accuracy for the Nusselt number. In other words, it seems, the model can predict well either the Nusselt number (that is, the derivative of the temperature at the wall) or the temperature field. This observation is, however, weaker in the case of the low Prandtl number, where the molecular heat transfer at the wall is dominant. If the focus of the study in an impinging jet case is the prediction of the temperature field, one might turn to the Reynolds analogy, however, if the focus is the prediction of temperature field, one might obtain better results with the AHFM-NRG model.

An additional observation is that the discrepancy of temperature prediction compared to the DNS, seems to rise towards the outflow for most of the models, except at the lowest Prandtl number results. At the lowest Prandtl number, there is a clear distinction between the momentum turbulence models. That is, the results with Launder-Sharma model the temperature discrepancy is decreasing, while with the v2-f model it is increasing. The similarity to the predictions of streamwise velocity component might be accidental.

3 Conclusions

In this deliverable the implementations of Manservigi's k_θ - ε_θ model and the AHFM-NRG models were tested on two forced convection cases: Kawamura's infinite channel and the planar impinging jet case of Duponcheel and Bartosiewicz. The performance of the two models is compared to other thermal turbulence models.

The two thermal momentum models perform reasonably well for prediction of temperature in the channel flow at the lowest Prandtl numbers. Nevertheless, the Manservigi's k_θ - ε_θ does not perform well for higher Prandtl numbers, for which it was not developed. The AHFM-NRG model performs well also over a range of Prandtl numbers. Both models, however, are not performing well in the case of predicting the temperature profile in the impinging jet case. Here, the Reynolds analogy would seem to be a better choice for temperature profile prediction. Nevertheless, for the prediction of the Nusselt number at the impinged wall, the AHFM-NRG model seems to be a better choice.

Of course, no tuning of any kind was performed for any model. This would of course significantly improve the predictions. Especially, since no consideration was given to the use of momentum turbulence models. Namely, both models were developed with a certain momentum turbulence model in mind and would need to be recalibrated for the use with different momentum models, to achieve better performance.

References

- [1] K. Hanjalić and B. Launder, *Modelling Turbulence in Engineering and the Environment*, Cambridge University Press, 2011.
- [2] A. Shams, A. De Santis and F. Roelofs, "An overview of the AHFM-NRG formulations for the accurate prediction of turbulent flow and heat transfer in low-Prandtl number flows," *Nuclear Engineering and Design*, vol. 355, p. 110342, December 2019.
- [3] W. M. Kays, "Turbulent Prandtl Number—Where Are We?," *Journal of Heat Transfer*, vol. 116, p. 284–295, May 1994.
- [4] S. Manservigi and F. Menghini, "CFD simulations in heavy liquid metal flows for square lattice bare rod bundle geometries with a four parameter heat transfer turbulence model," *Nuclear Engineering and Design*, vol. 295, p. 251–260, December 2015.
- [5] H. Kawamura, H. Abe and K. Shingai, "DNS of turbulence and heat transport in a channel flow with different Reynolds and Prandtl numbers and boundary conditions," in *3rd Int. Symp. on Turbulence, Heat and Mass Transfer*, 2000.
- [6] M. Duponcheel and Y. Bartosiewicz, "Direct Numerical Simulation of Turbulent Heat Transfer at Low Prandtl Numbers in Planar Impinging Jets," *International Journal of Heat and Mass Transfer*, vol. 173, p. 121179, July 2021.
- [7] H. Kawamura, *DNS Database of Wall Turbulence and Heat Transfer*, 2021.
- [8] D. Cerroni, S. Manservigi and F. Menghini, "Numerical Validation of a κ - ω - $\kappa\theta$ - $\omega\theta$ Heattransfer Turbulence Model for Low Prandtl Number Fluids," in *11th World Congress on Computational Mechanics (WCCM XI), 5th European Conference on Computational Mechanics (ECCM V), 6th European Conference on Computational Fluid Dynamics (ECFD VI)*, 2014.
- [9] S. Kenjereš, S. B. Gunarjo and K. Hanjalić, "Contribution to elliptic relaxation modelling of turbulent natural and mixed convection," *International Journal of Heat and Fluid Flow*, vol. 26, p. 569–586, August 2005.
- [10] A. De Santis, A. Villa Ortiz, A. Shams and L. Koloszar, "Modelling of a planar impinging jet at unity, moderate and low Prandtl number: Assessment of advanced RANS closures," *Annals of Nuclear Energy*, vol. 129, p. 125–145, July 2019.

Appendix A: Derivation of equations

Starting with equations for incompressible flow

$$\frac{\partial v_i}{\partial x_i} = 0. \quad (13)$$

The equation for conservation of momentum

$$\rho \frac{\partial v_i}{\partial t} + \rho v_j \frac{\partial v_i}{\partial x_j} = -\frac{\partial p}{\partial x_i} + \mu \frac{\partial^2 v_i}{\partial x_j \partial x_j}. \quad (14)$$

And the equation of conservation of energy

$$\frac{\partial T}{\partial t} + v_i \frac{\partial T}{\partial x_i} = \alpha \frac{\partial^2 T}{\partial x_i \partial x_i}. \quad (15)$$

We introduce the following substitutions, where we also consider the linear pressure drop and linear temperature increase through the domain:

$$p = p_1 - \frac{\Delta p}{h} x_1, \quad (16)$$

$$T = T_0 - T_1 + \frac{\Delta T}{h} x_1. \quad (17)$$

It is implicitly assumed that the boundary condition for p and T_1 , as well as for v_i is periodic in x and z directions.

The momentum equation is then rewritten as

$$\rho \frac{\partial v_i}{\partial t} + \rho v_j \frac{\partial v_i}{\partial x_j} = -\frac{\partial p_1}{\partial x_i} + \mu \frac{\partial^2 v_i}{\partial x_j \partial x_j} + \frac{\Delta p}{h} \delta_{i1} \quad (18)$$

and the equation for conservation of energy as

$$\frac{\partial T_1}{\partial t} + v_i \frac{\partial T_1}{\partial x_i} = \alpha \frac{\partial^2 T_1}{\partial x_i \partial x_i} + \frac{v_1 \Delta T}{h} \quad (19)$$

The Δp in the momentum equation is a free parameter that can be used to achieve either the desired bulk velocity or friction velocity. In first case the `meanVelocityForce` is used in OpenFOAM where this parameter is actively adjusted at each time step. In the latter case, where a desired bulk velocity is to be achieved, $\Delta p = \rho u_\tau^2$.

From integral power balance at steady state ($P = \dot{m} c_p (T_o - T_i)$ or integrating equation (15) over the volume of the part of the channel) we can obtain the relation between the ΔT and total power P with which a part of the channel of dimensions $L_x \times L_y \times L_z$ (where $L_y = 2h$) is heated:

$$\Delta T = \frac{q}{\rho v_B c_p}, \quad (20)$$

where q is $P/(2L_x L_z)$ and v_B is the bulk velocity of the flow.

From the boundary condition for thermal equation ($P = -\oint \mathbf{j} \cdot d\mathbf{S}$, where $\mathbf{j}$ is the thermal flux; note that the streamwise direction flux is cancelled out) one can write the boundary condition in steady state (that is, the flux over the walls is constant over both walls) used in OpenFOAM as

$$n_j \frac{\partial T_1}{\partial x_j} = -\frac{q}{\lambda}, \quad (21)$$

where n_j is the outward facing normal vector at the lower and upper walls.

To obtain the equations used by Kawamura [5], the dimensionless variables defined implicitly (with * or + superscripts), are

$$t = \frac{h}{u_\tau} t^+, \quad (22)$$

$$x_i = h x_i^*, \quad (23)$$

$$v_i = u_\tau u_i^+, \quad (24)$$

$$p = \rho u_\tau^2 p^+ - \rho u_\tau^2 x_1, \quad (25)$$

$$T = T_0 - T_d \theta^+ + \frac{q}{\rho v_B c_p h} x_1. \quad (26)$$

Inserting into equations (13), (14) and (15) one obtains

$$\frac{\partial u_i^+}{\partial x_i^*} = 0, \quad (27)$$

$$\frac{\partial u_i^+}{\partial t^+} + u_j^+ \frac{\partial u_i^+}{\partial x_j^*} = -\frac{\partial p^+}{\partial x_i^*} + \frac{1}{\text{Re}_\tau} \frac{\partial^2 u_i^+}{\partial x_j^* \partial x_j^*} + 1, \quad (28)$$

$$\frac{\partial \theta^+}{\partial t^+} + u_j^+ \frac{\partial \theta^+}{\partial x_j^*} = \frac{1}{\text{Re}_\tau \text{Pr}} \frac{\partial^2 \theta^+}{\partial x_j^* \partial x_j^*} + \frac{q}{\rho u_\tau c_p T_d} \frac{u_1^+}{u_B}, \quad (29)$$

where $u_B = v_B/u_\tau$. Kawamura sets

$$T_d = \frac{q}{\rho u_\tau c_p}, \quad (30)$$

which also implies that the heat flux at the wall

$$n_j \frac{\partial \theta^+}{\partial x_j^*} = -\frac{q}{\lambda T_d} h = -\text{Pr} \text{Re}_\tau. \quad (31)$$

Written with wall units, this is equivalent to

$$n_j \frac{\partial \theta^+}{\partial x_j^+} = -\text{Pr}, \quad (32)$$

or at the lower wall

$$\frac{\partial \theta^+}{\partial y^+} = \text{Pr}. \quad (33)$$

Although this heat flux is implied, the boundary condition in the DNS is $\theta^+ = 0$.

Equation (29) has the final form

$$\frac{\partial \theta^+}{\partial t^+} + u_j^+ \frac{\partial \theta^+}{\partial x_j^*} = \frac{1}{\text{Re}_\tau \text{Pr}} \frac{\partial^2 \theta^+}{\partial x_j^* \partial x_j^*} + \frac{u_1^+}{u_B}. \quad (34)$$

Appendix B: Momentum turbulence models comparison for $Re_\tau = 640$

In this appendix the comparison figures, similar to the comparisons in section 1.3.1 are presented. The conclusions are presented in that section.

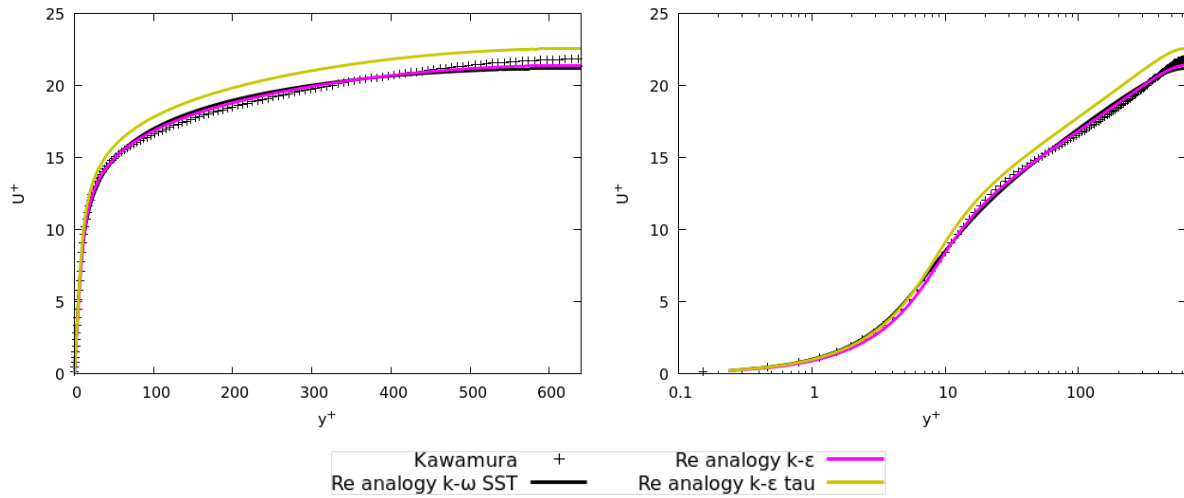


Figure 22: Comparison of average streamwise velocity component between Launder-Sharma $k-\epsilon$ model with bulk velocity source term, Launder-Sharma $k-\epsilon$ model with tau implementation of source term and $k-\omega$ SST model with bulk velocity source term for the case with $Re_\tau = 640$.

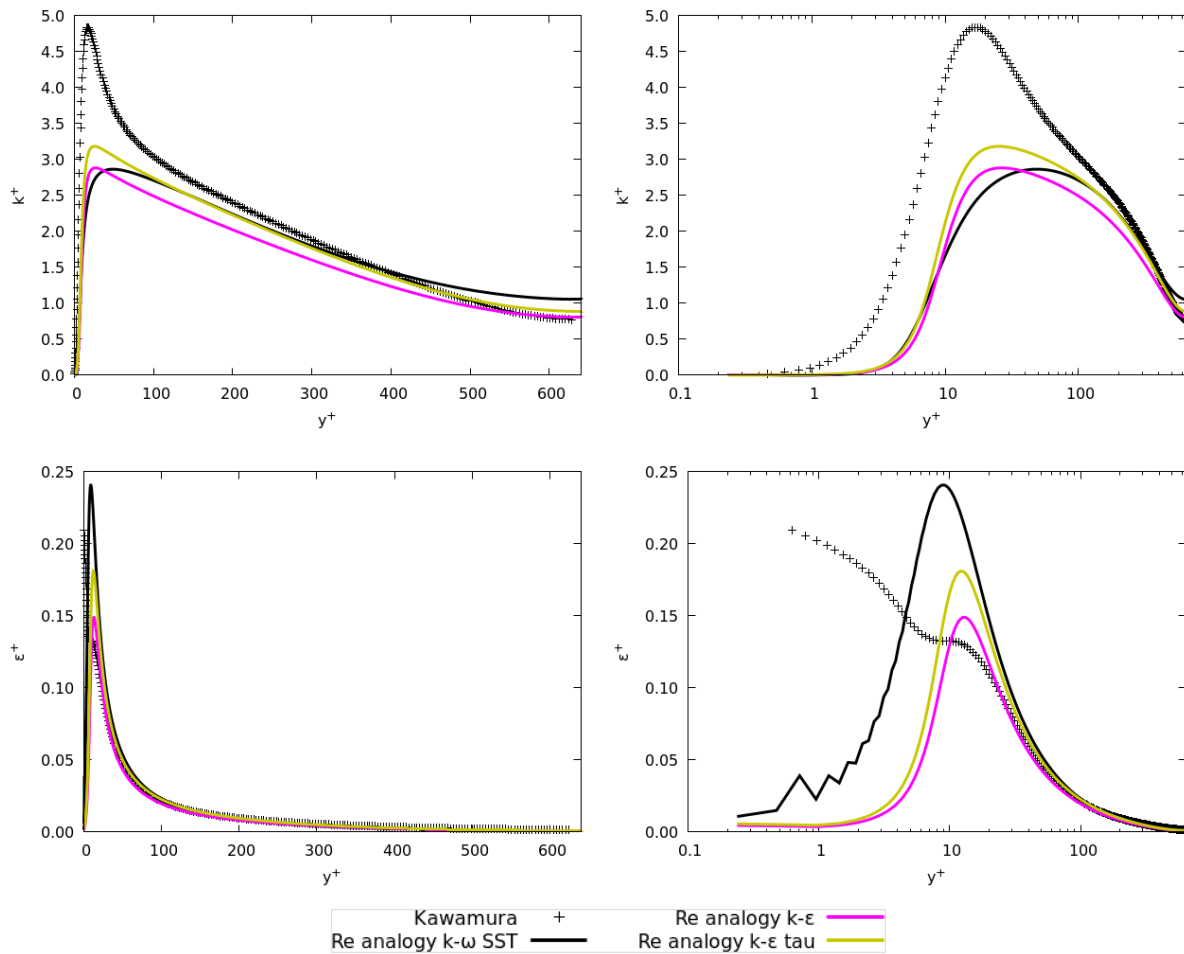


Figure 23: Comparison of turbulence kinetic energy (top) and dissipation of turbulence kinetic energy (bottom) between Launder-Sharma $k-\epsilon$ model with bulk velocity source term, Launder-Sharma $k-\epsilon$ model with tau implementation of source term and $k-\omega$ SST model with bulk velocity source term for the case with $Re_\tau = 640$.

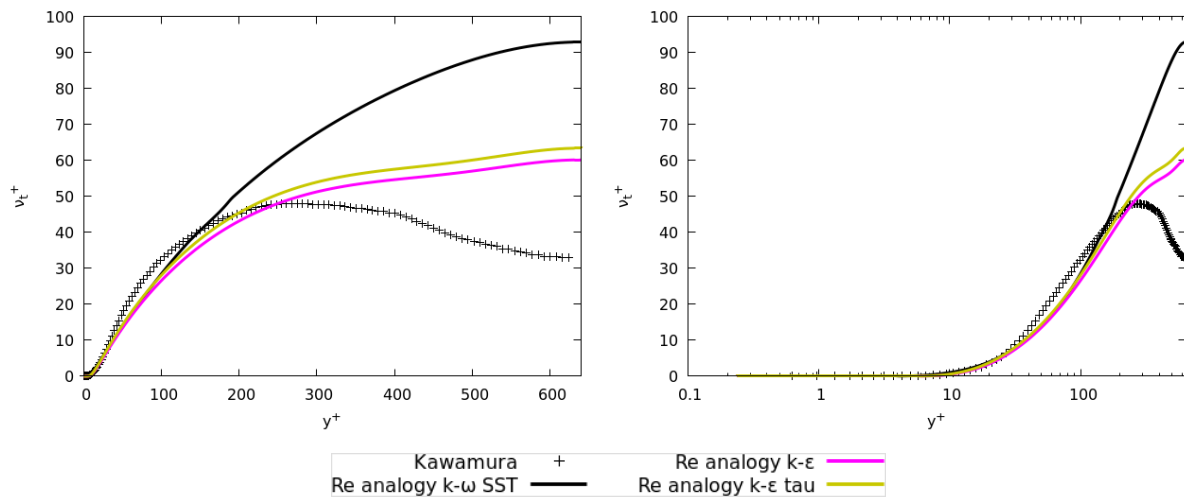


Figure 24: Comparison of turbulence eddy viscosity between Launder-Sharma k - ε model with bulk velocity source term, Launder-Sharma k - ε model with tau implementation of source term and k - ω SST model with bulk velocity source term for the case with $Re_\tau = 640$.

Appendix C: Additional figures to comparison of thermal turbulence models

Temperature

$$Re_\tau = 395$$

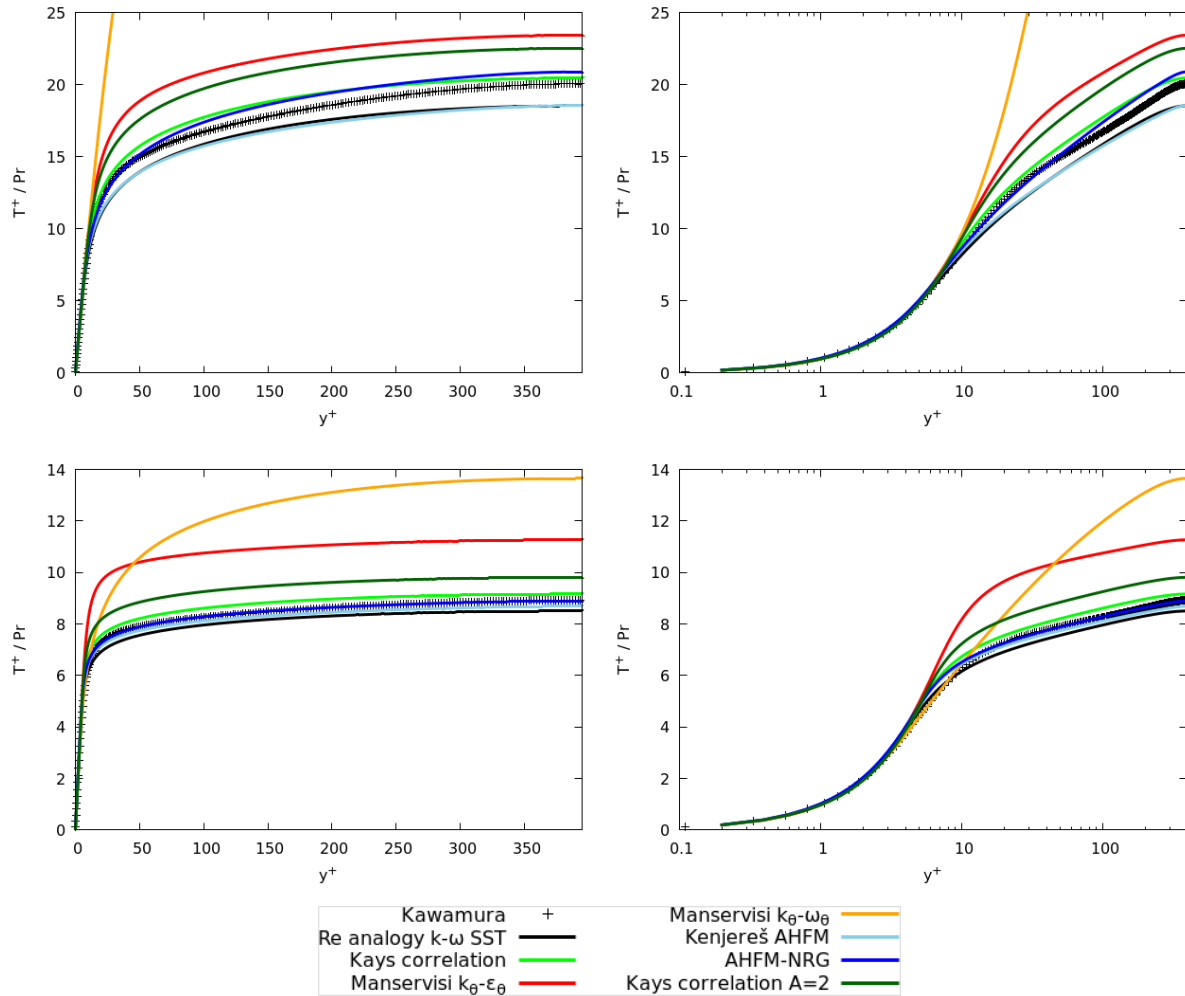


Figure 25: Temperature distribution predicted by different thermal turbulence models for $Pr = 1$ (top) and $Pr = 5$ (bottom) and $Re_\tau = 395$.

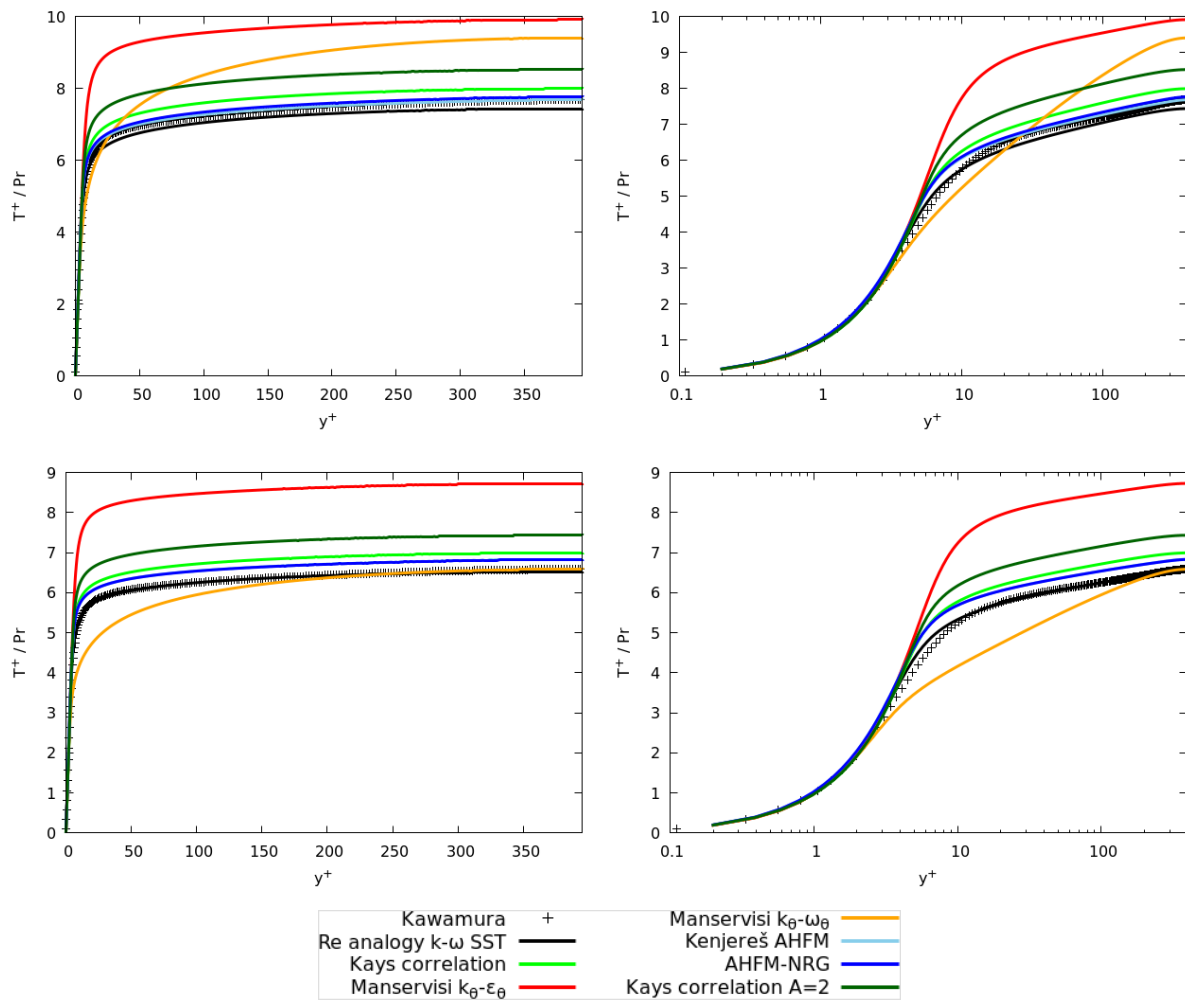


Figure 26: Temperature distribution predicted by different thermal turbulence models for $Pr = 7$ (top) and $Pr = 10$ (bottom) and $Re_\tau = 395$.

$Re_\tau = 640$

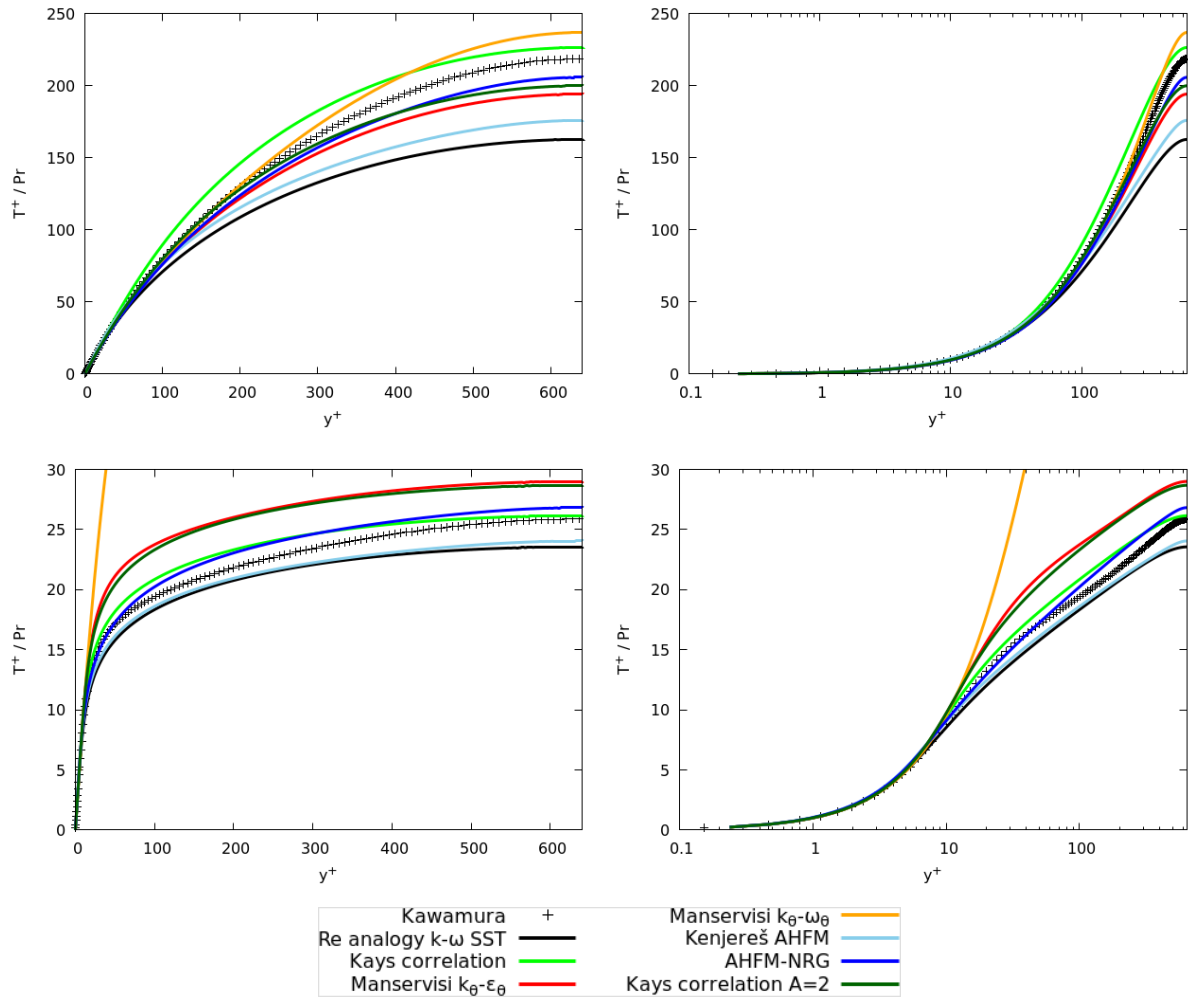


Figure 27: Temperature distribution predicted by different thermal turbulence models for $Pr = 0.025$ (top) and $Pr = 0.71$ (bottom) and $Re_\tau = 640$.

Thermal fluctuations

The convention with $k_\theta^+ = \frac{\langle (\theta^+)'^2 \rangle}{2}$ is used. Only predictions of models that calculate k_θ are shown.

$Re_\tau = 395$

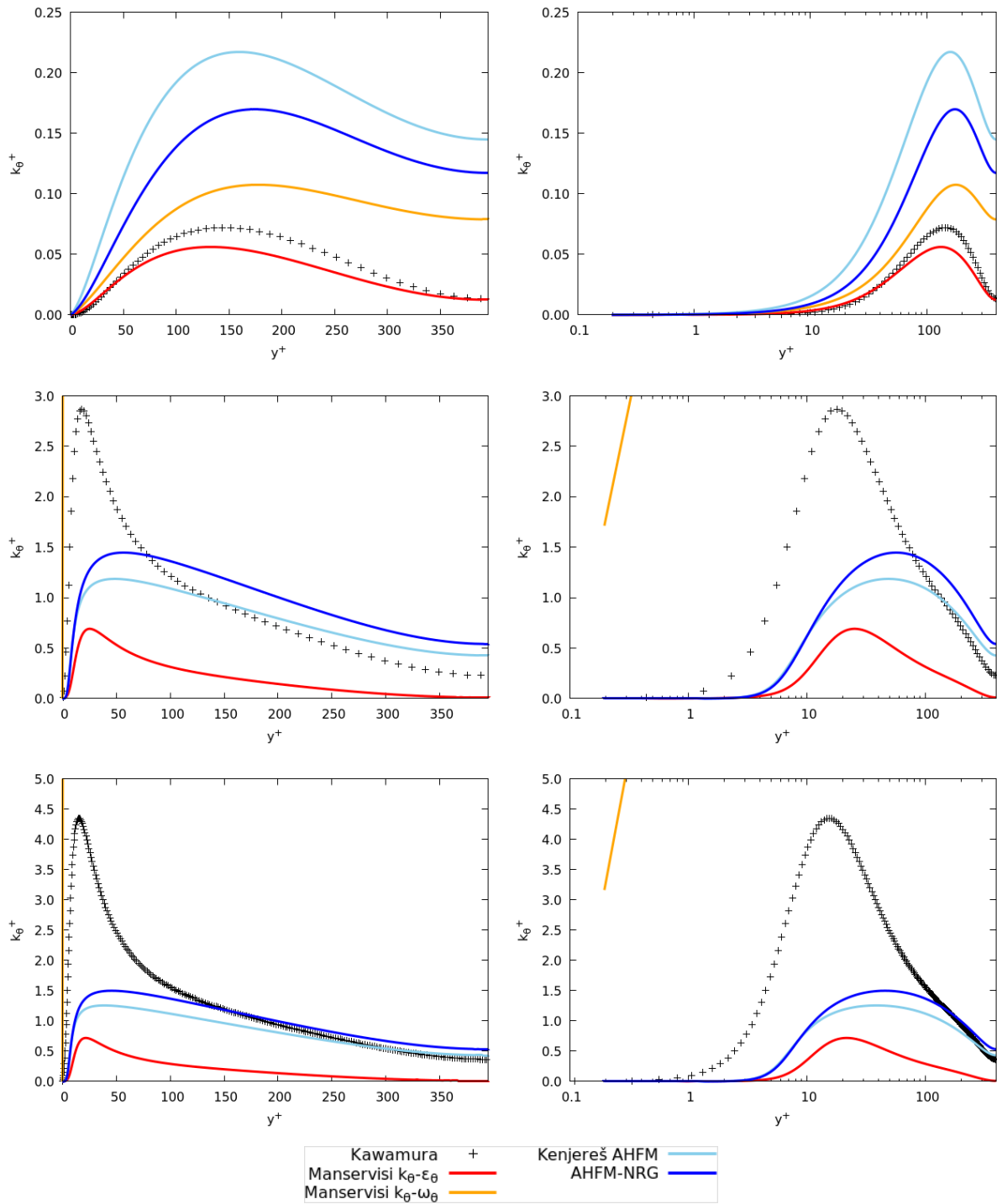


Figure 28: Thermal fluctuations for $Pr = 0.025$ (top), $Pr = 0.71$ (middle) and $Pr = 1$ (bottom) at $Re_\tau = 395$.

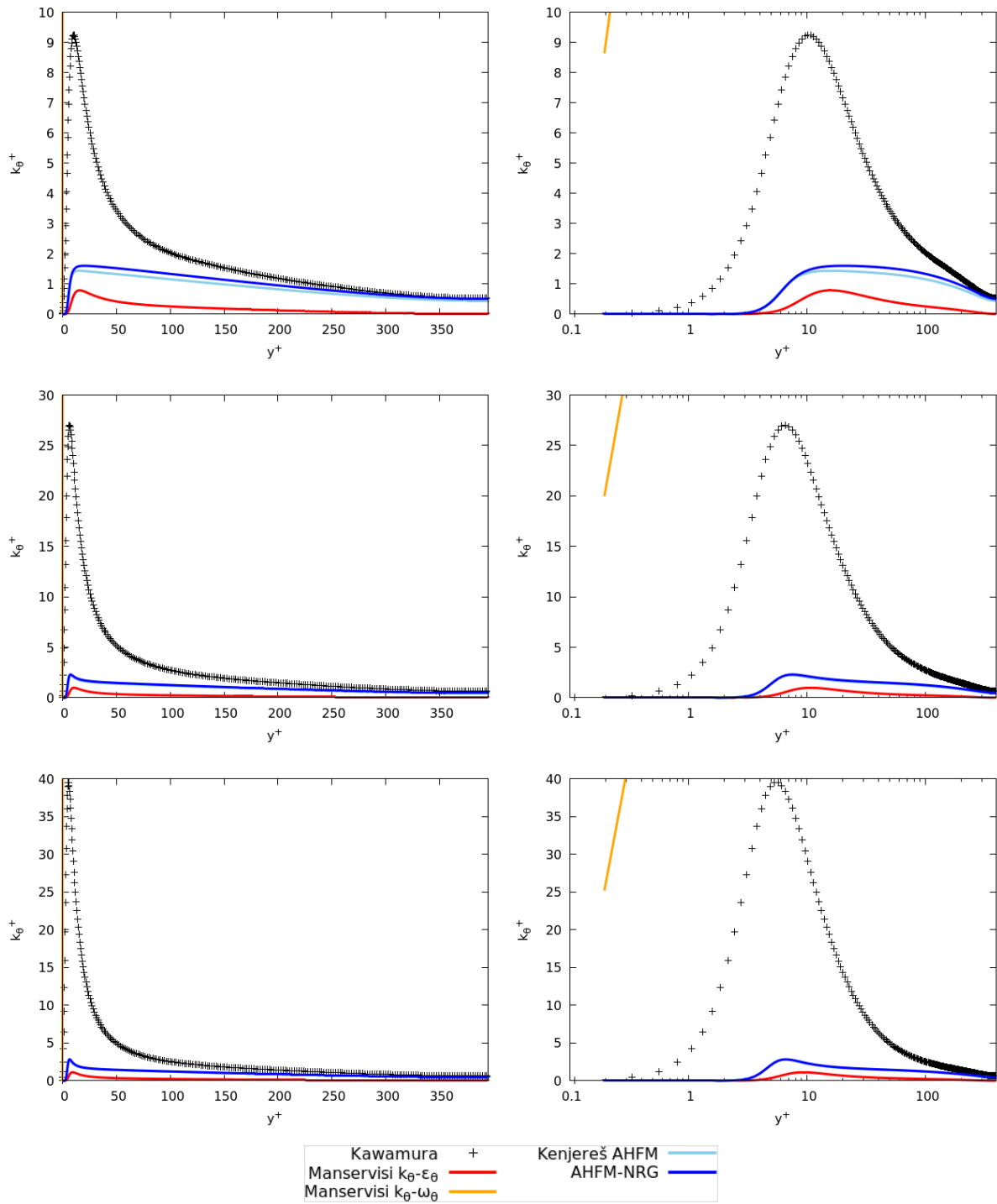


Figure 29: Thermal fluctuations for $Pr = 2$ (top), $Pr = 5$ (middle) and $Pr = 7$ (bottom) at $Re_\tau = 395$.

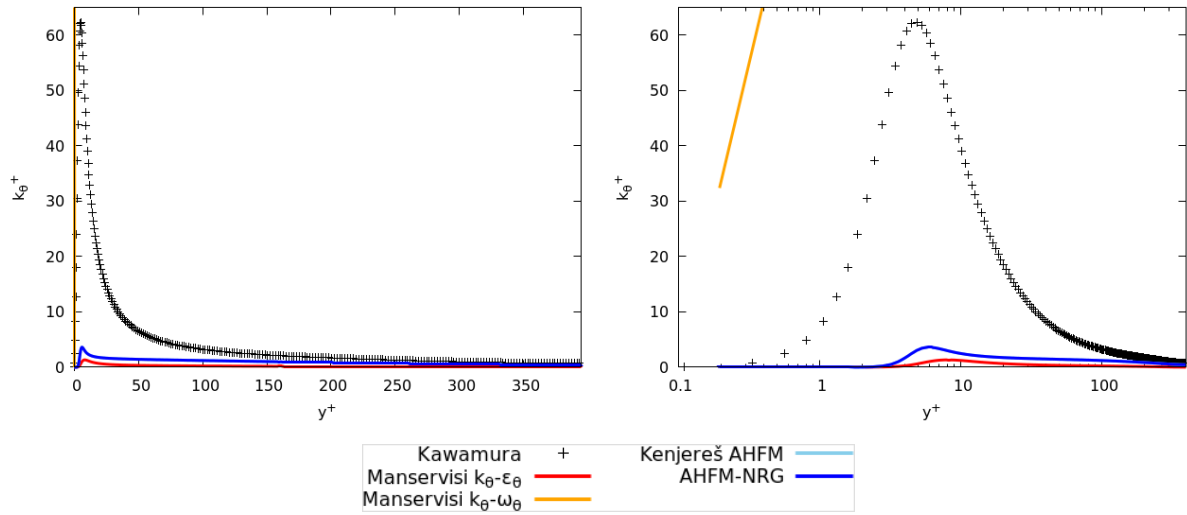


Figure 30: Thermal fluctuations for $Pr = 10$ at $Re_\tau = 395$.

$Re_\tau = 640$

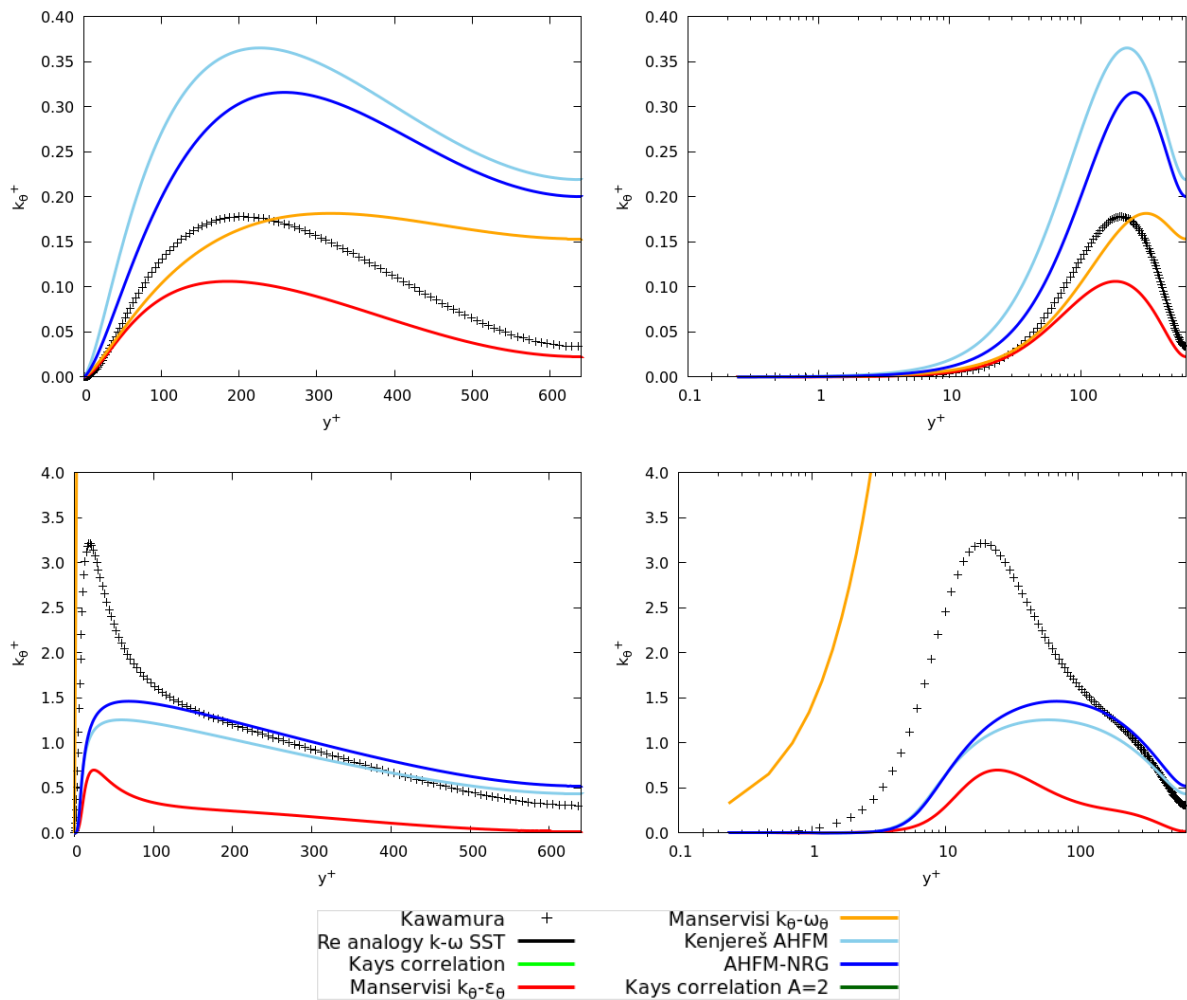


Figure 31: Thermal fluctuations for $Pr = 0.025$ (top), $Pr = 0.71$ (middle) at $Re_\tau = 640$.

Thermal turbulent diffusion

Note that the quantity $Pr \alpha_t^+ = \alpha_t / \alpha$, shown in the figures, is the ratio between turbulent and molecular thermal diffusivities.

$Re_\tau = 395$

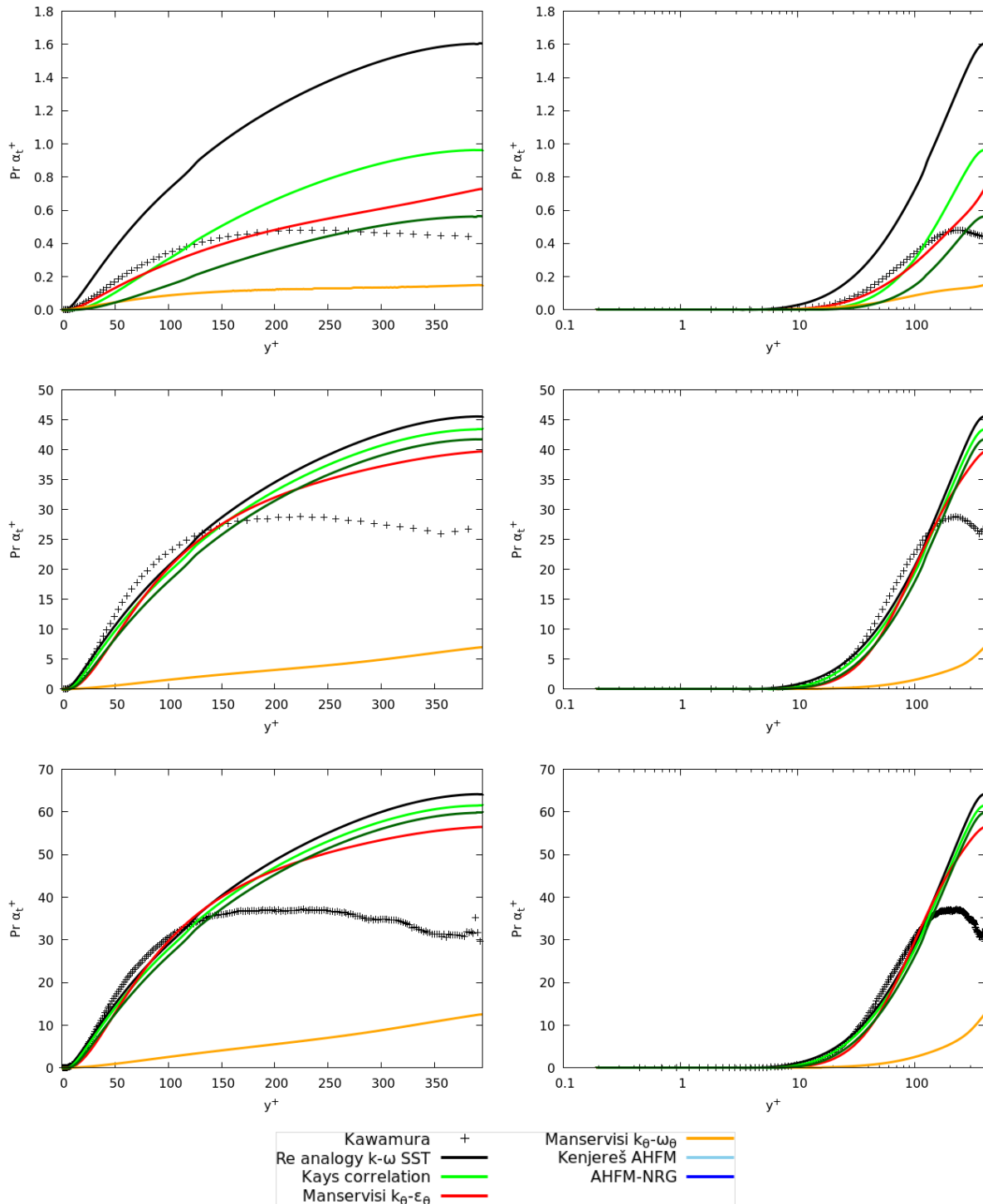


Figure 32: Turbulent thermal diffusion for $Pr = 0.025$ (top), $Pr = 0.71$ (middle) and $Pr = 1$ (bottom) at $Re_\tau = 395$.

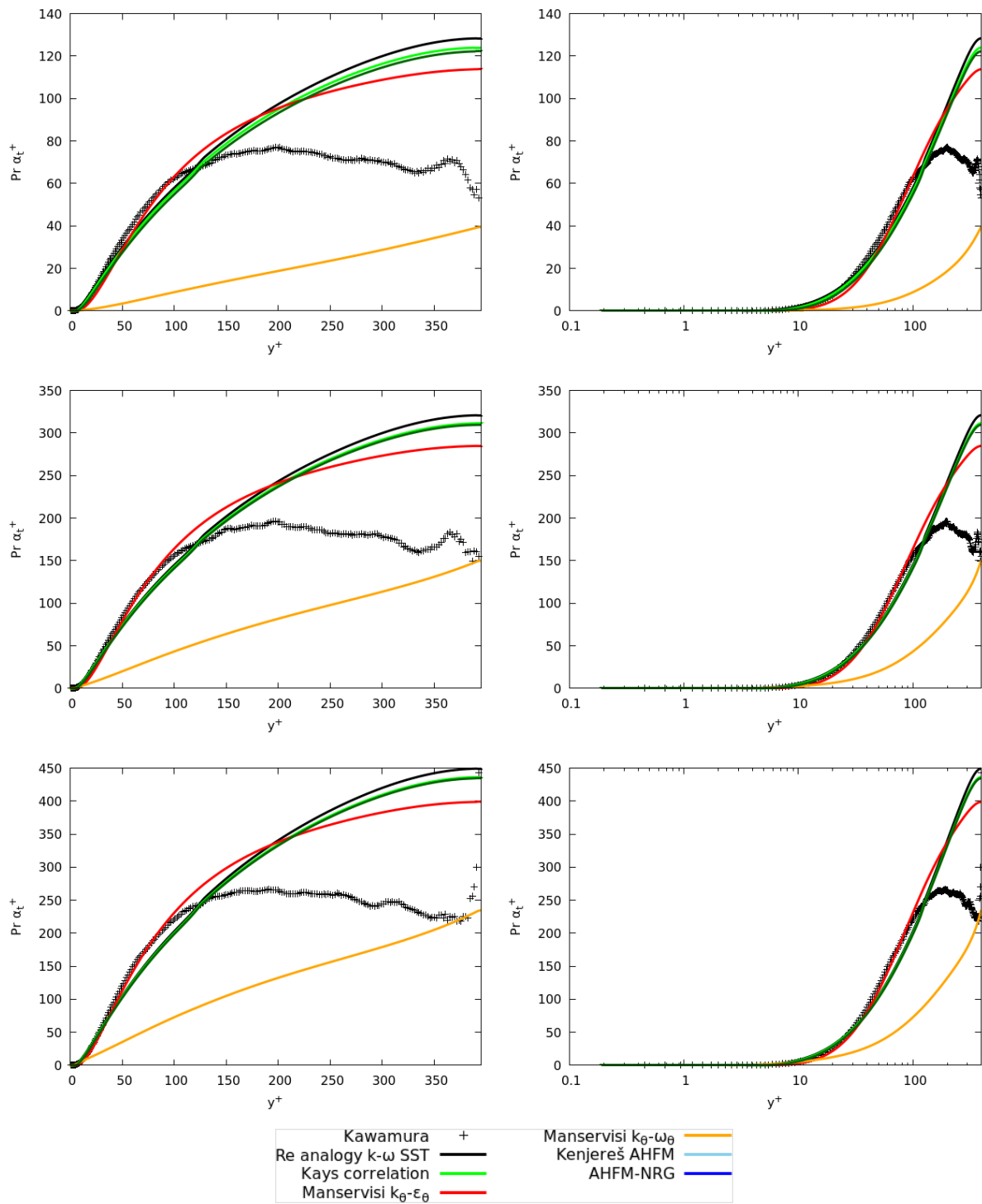


Figure 33: Turbulent thermal diffusion for $Pr = 2$ (top), $Pr = 5$ (middle) and $Pr = 7$ (bottom) at $Re_\tau = 395$.

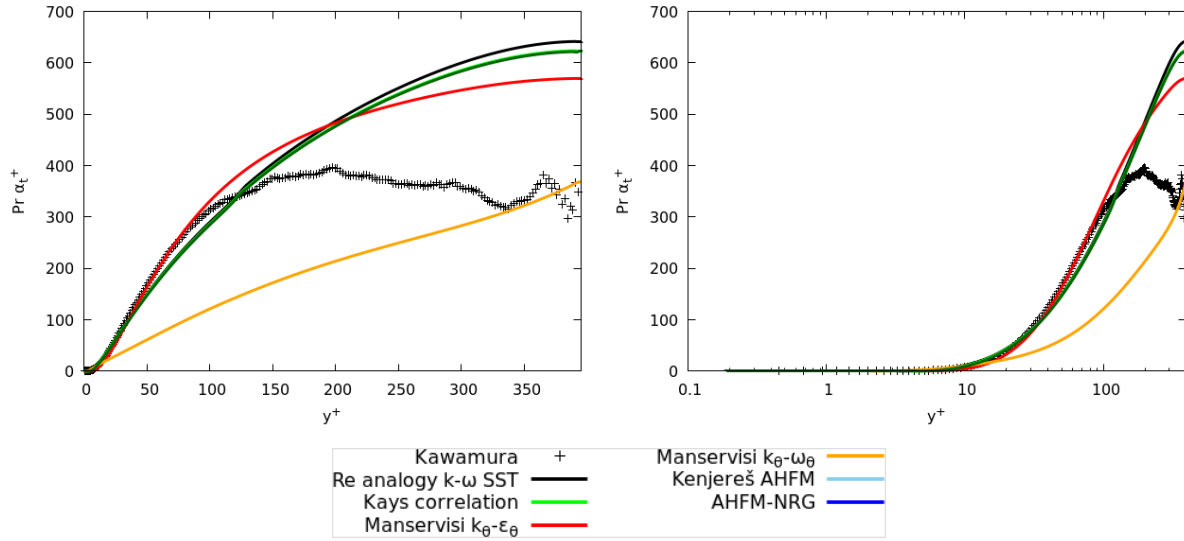


Figure 34: Turbulent thermal diffusion for $Pr = 10$ at $Re_\tau = 395$.

$Re_\tau = 640$

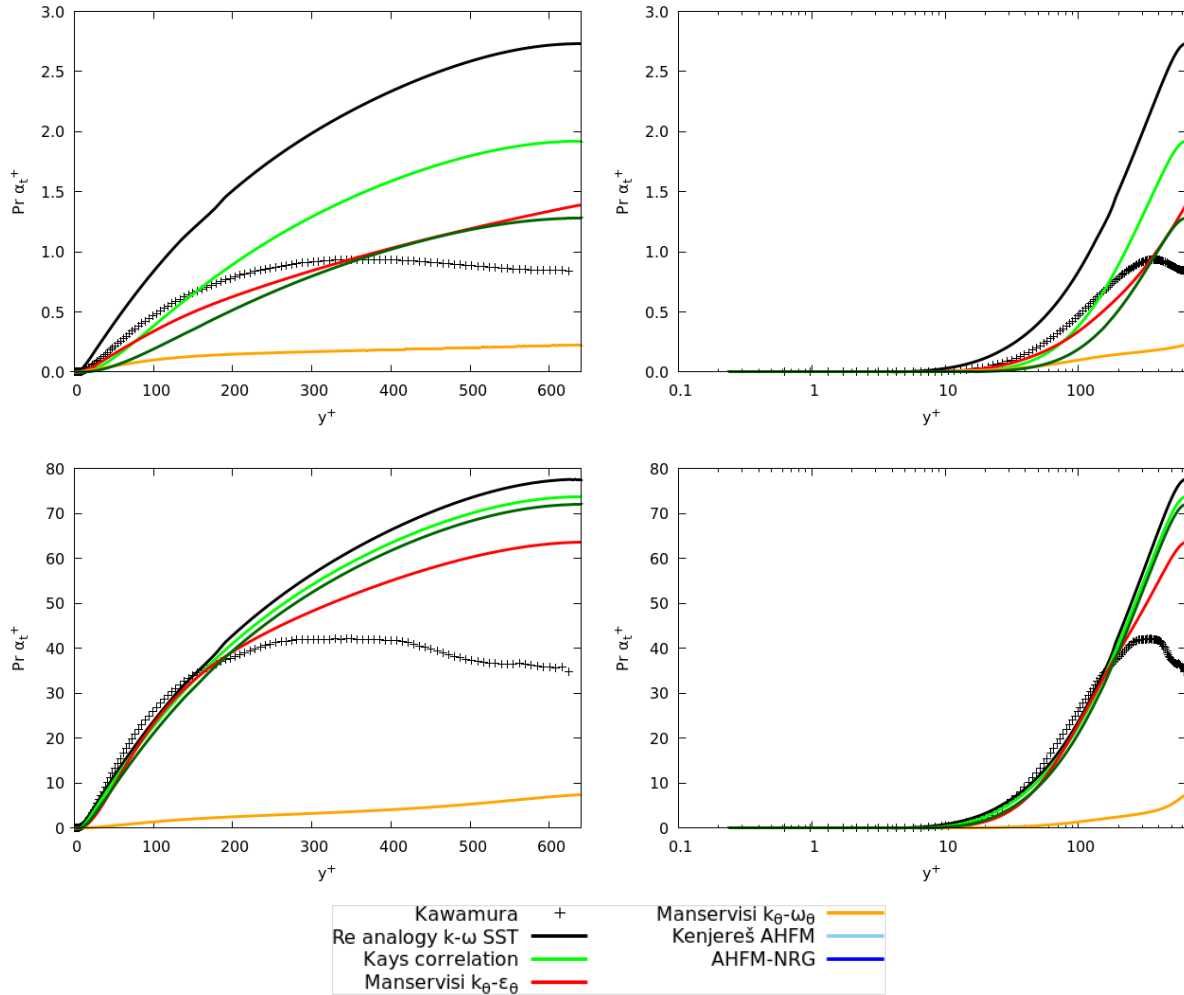


Figure 35: Turbulent thermal diffusion for $Pr = 0.025$ (top) and $Pr = 0.71$ (bottom) at $Re_\tau = 640$.

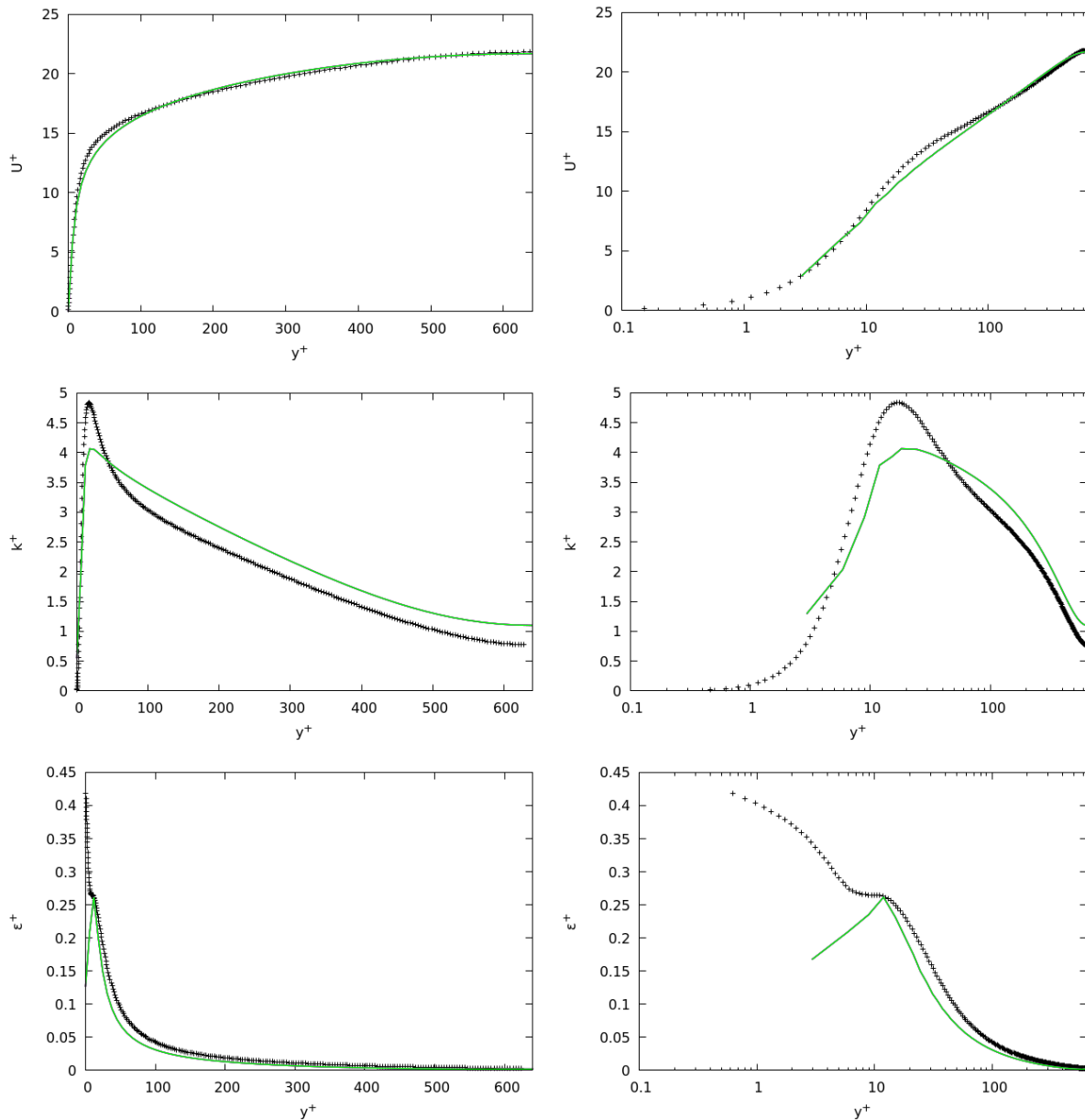
Appendix D: Mesh convergence study

In this Appendix, the results of the mesh convergence study are presented.

Mesh convergence in the streamwise direction

Mesh convergence study in the streamwise direction was only performed for the higher friction Reynolds number of 640. For a mesh with 50 cells in the wall normal direction in the half-channel four meshes were tested, which give virtually identical results. For the mesh with 300 cells in the wall direction in the half-channel, only one mesh was tested. The results are identical, so the 50 cells in the streamwise direction was found as adequate.

$N = 50$



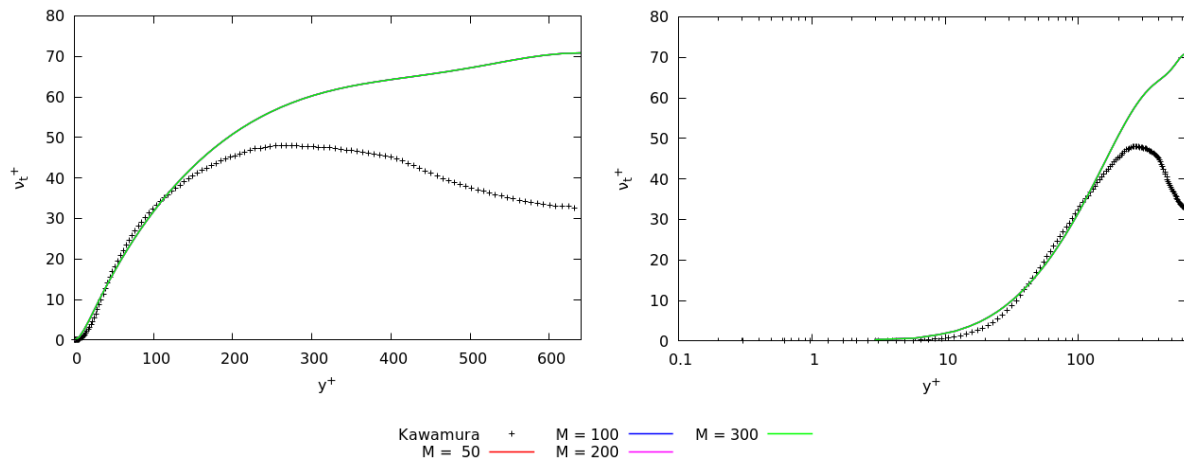


Figure 36: Comparison of various momentum and momentum turbulence variables from meshes with different number of cells in the streamwise direction.

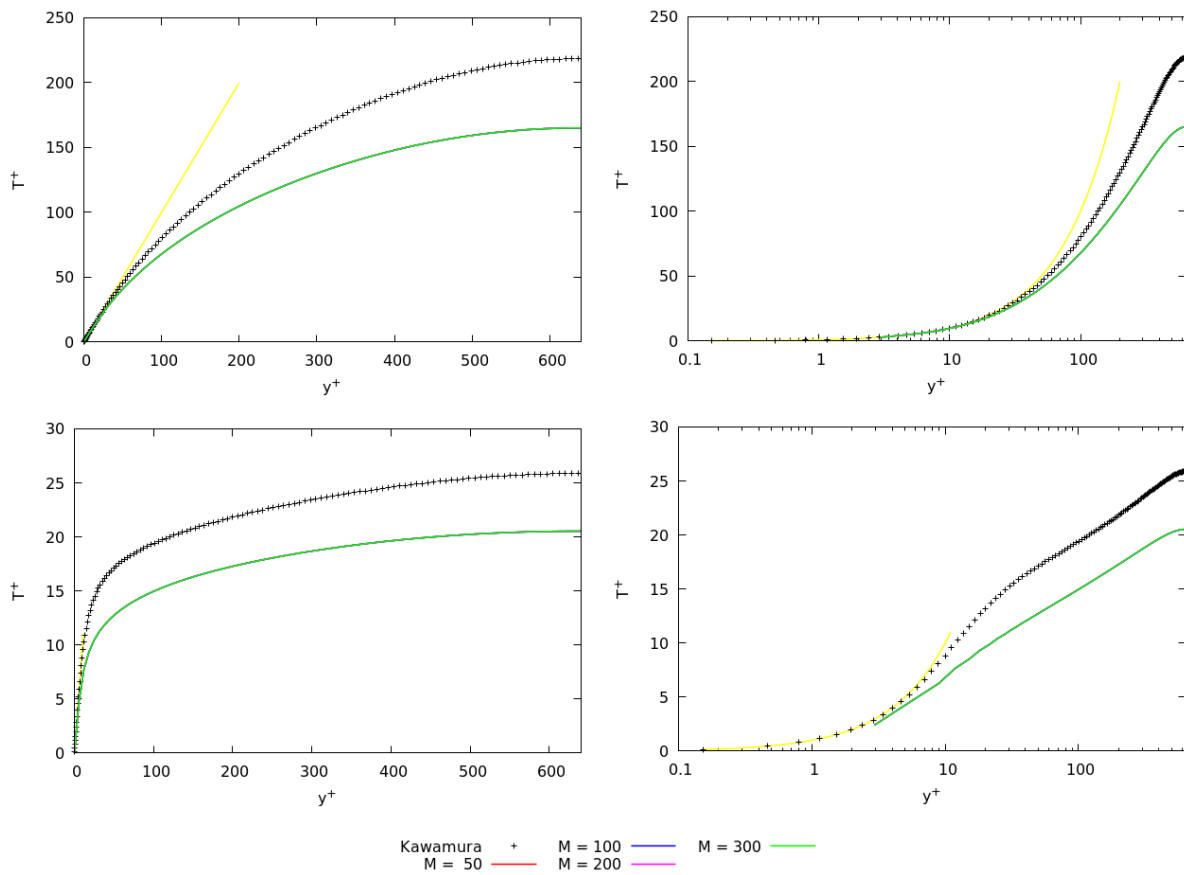


Figure 37: Temperature for Prandtl numbers 0.025 and 0.71 obtained with various number of cells in the streamwise direction.

$N = 300$

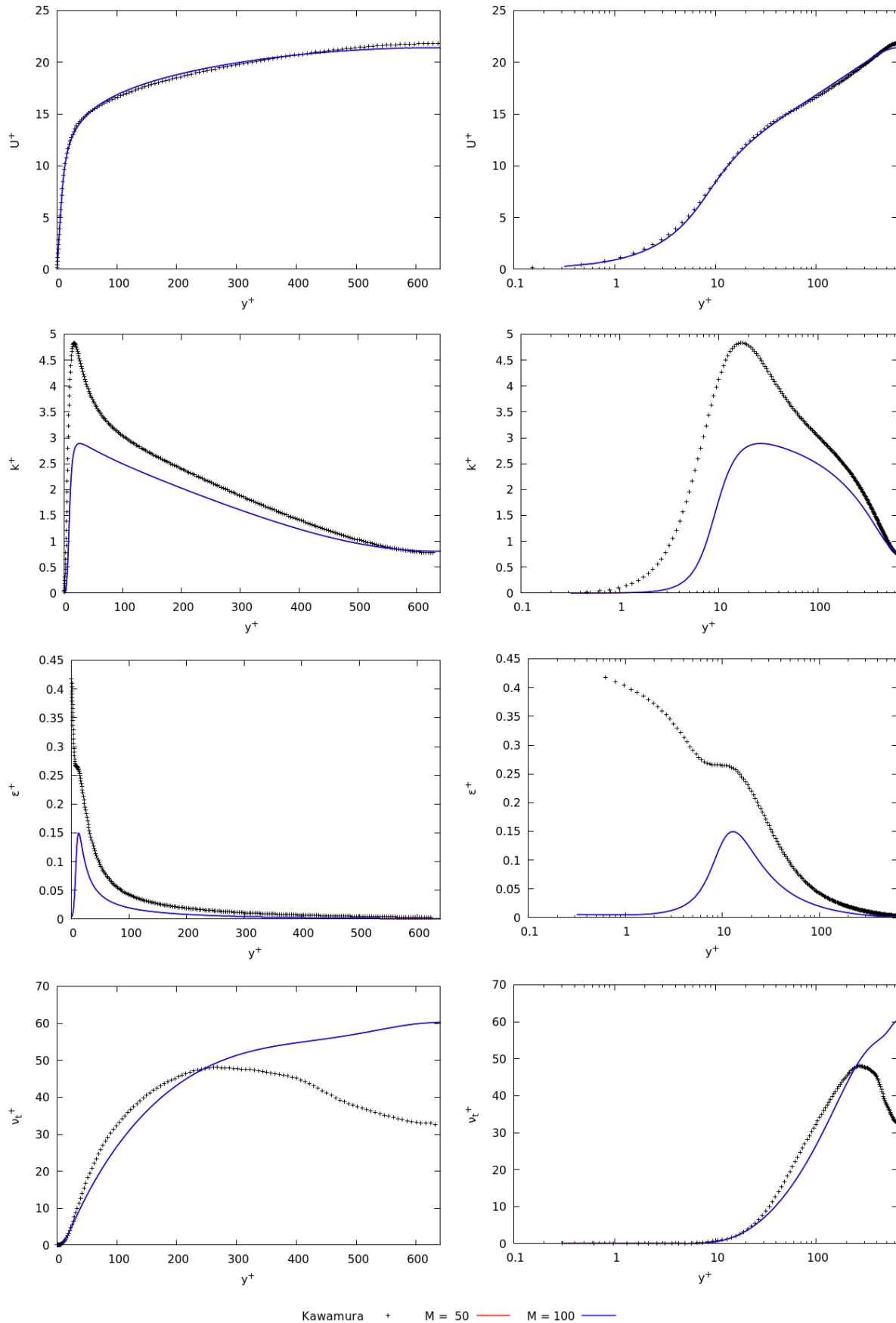


Figure 38: Comparison of various momentum and momentum turbulence variables from the two meshes with different number of cells in the streamwise direction.

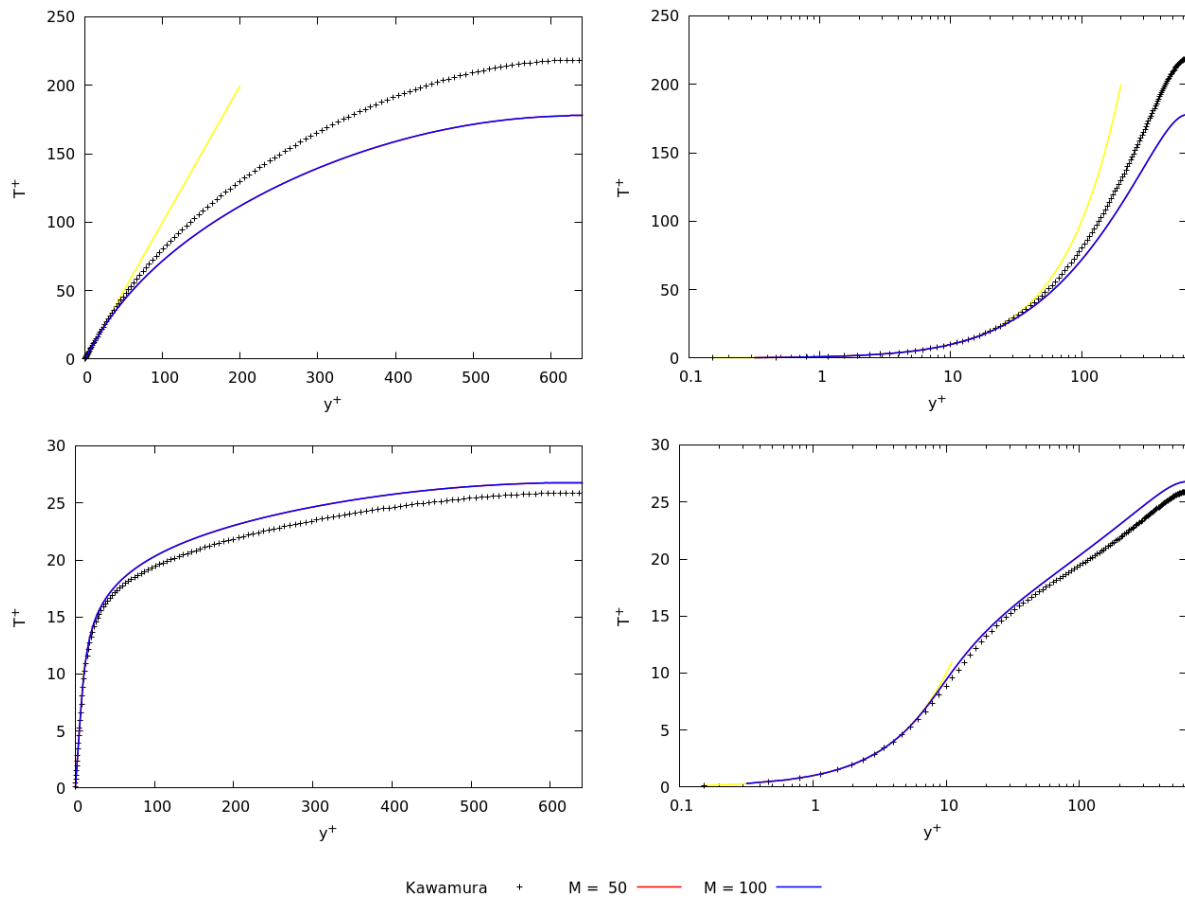


Figure 39: Temperature for Prandtl numbers 0.025 and 0.71 obtained with various number of cells in the streamwise direction.

Mesh convergence for $k-\omega$ SST model and comparison with LS $k-\epsilon$ model results

Based on the results presented here, the mesh with 300 cells in the wall normal direction of half-channel was selected for the friction Reynolds number 395, and mesh with 400 cells in the wall normal direction of half-channel was selected for the friction Reynolds number 640.

3.1.1 $Re_\tau = 395$

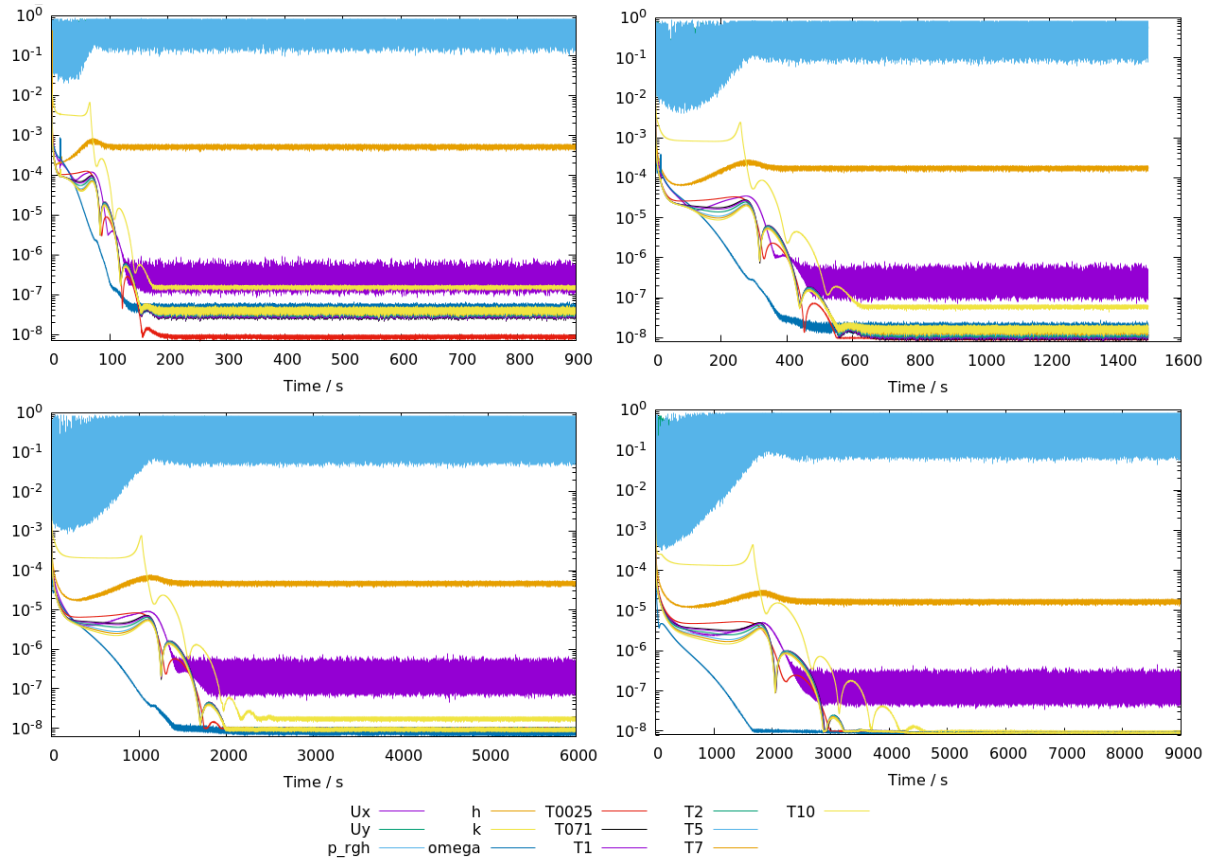
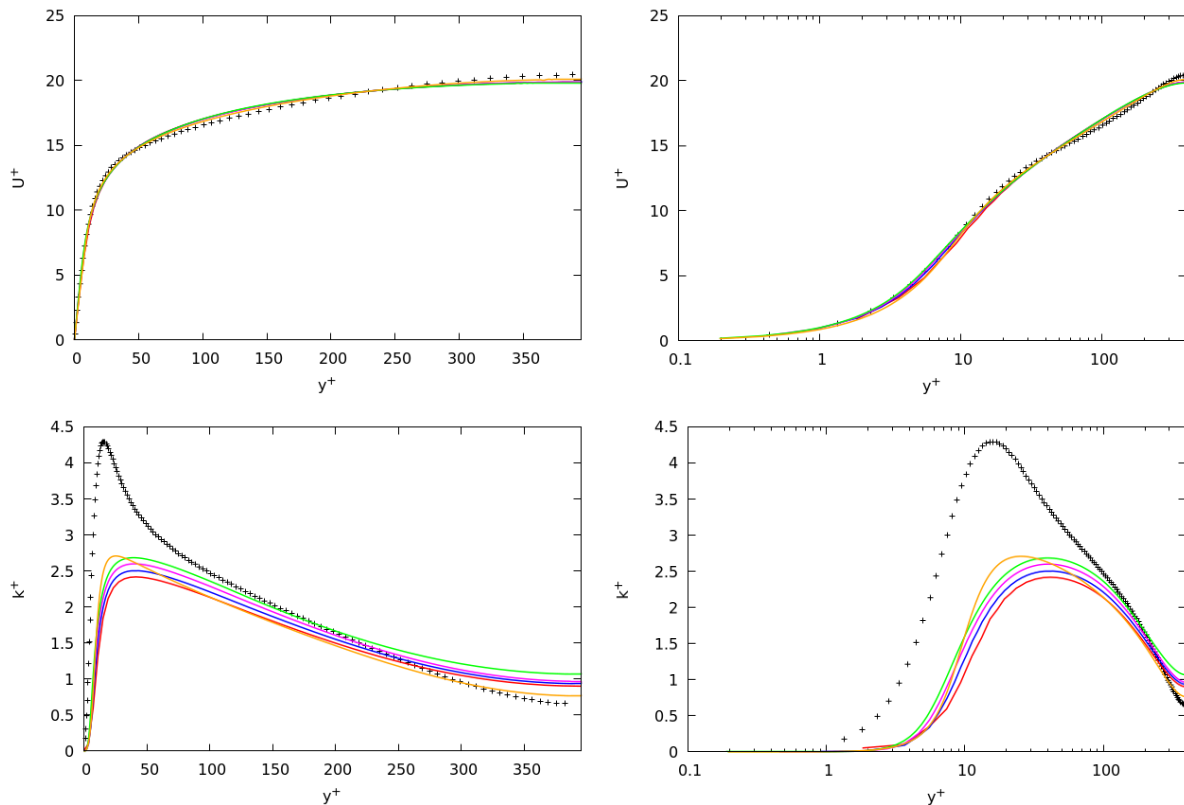


Figure 40: Residuals obtained with meshes $N = 50$ (top left), $N = 100$ (top right), $N = 200$ (bottom left) and $N = 300$ (bottom right) with $k-\omega$ SST model and Reynolds analogy thermal model at $Re_\tau = 395$.



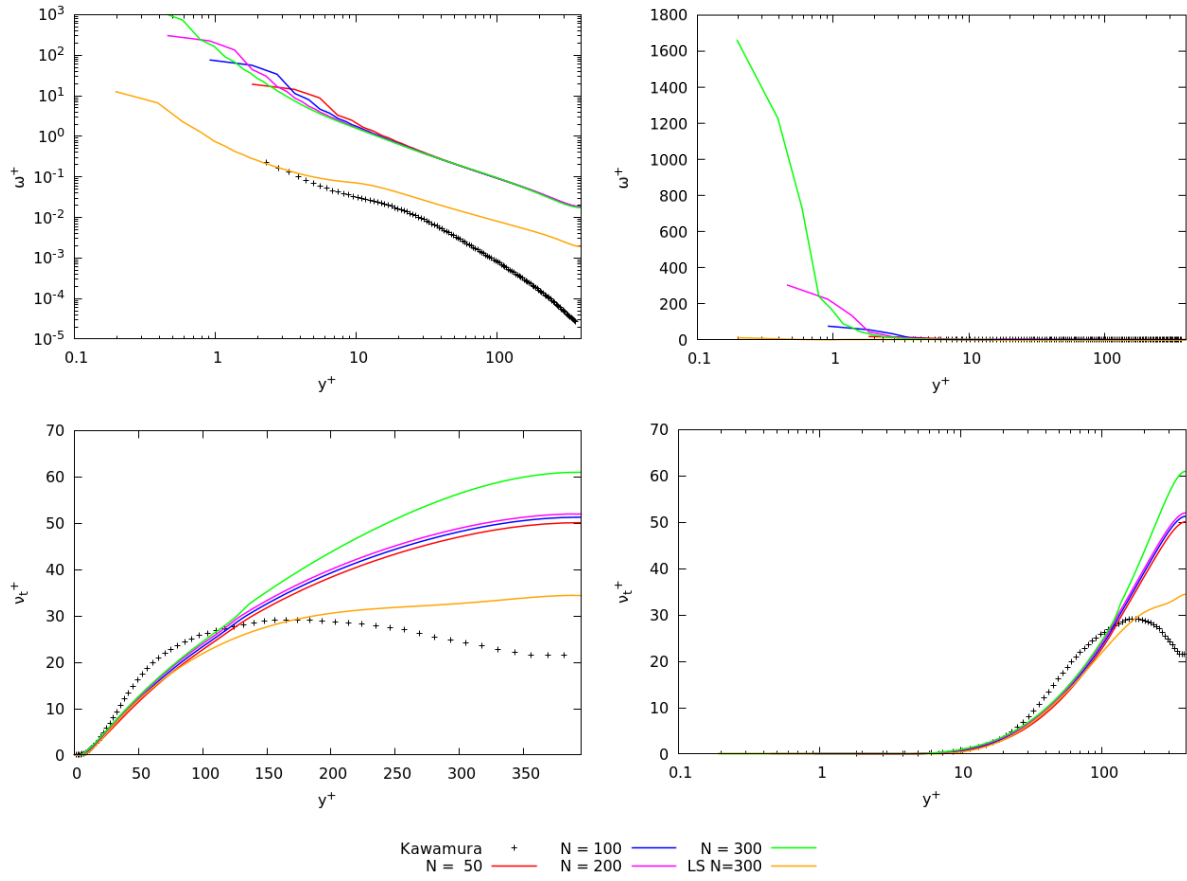
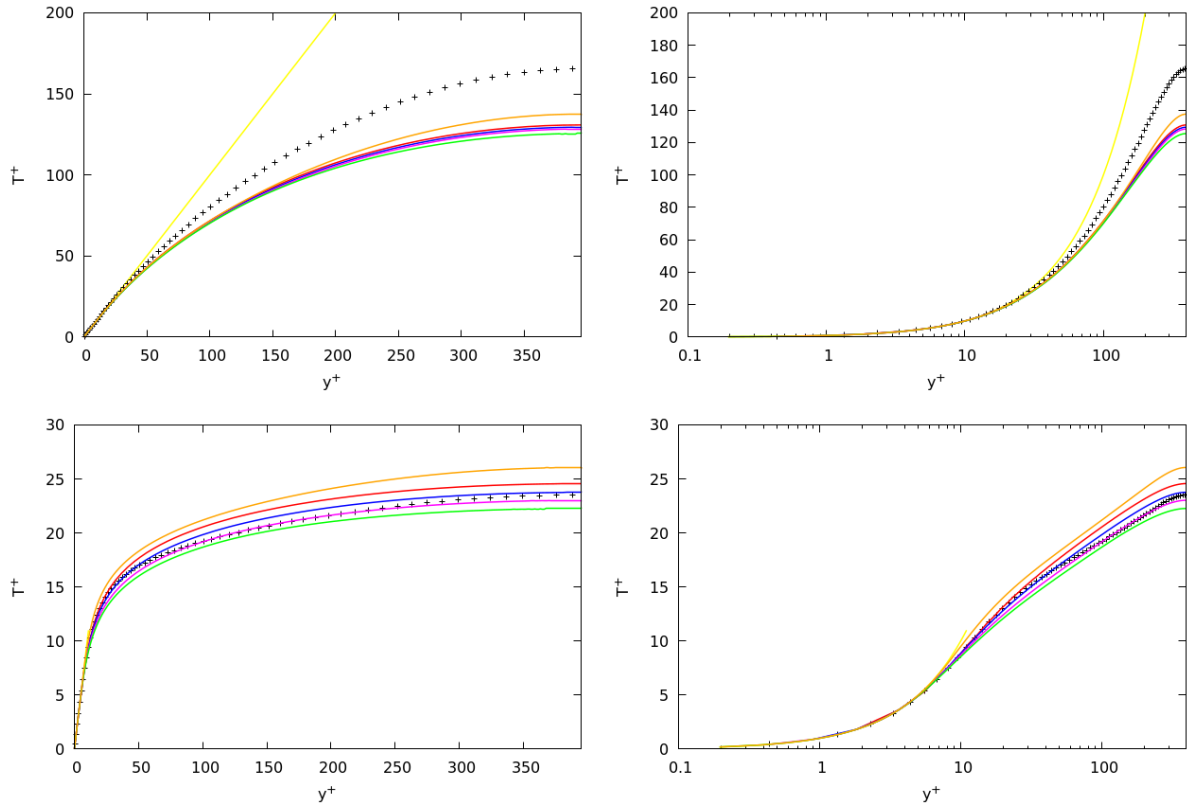


Figure 41: Comparison of momentum and turbulent variables between the four meshes for $k\text{-}\omega$ SST model at $Re_\tau = 395$. LS $N = 300$ marks results obtained with Launder-Sharma $k\text{-}\varepsilon$ model on the finest mesh.



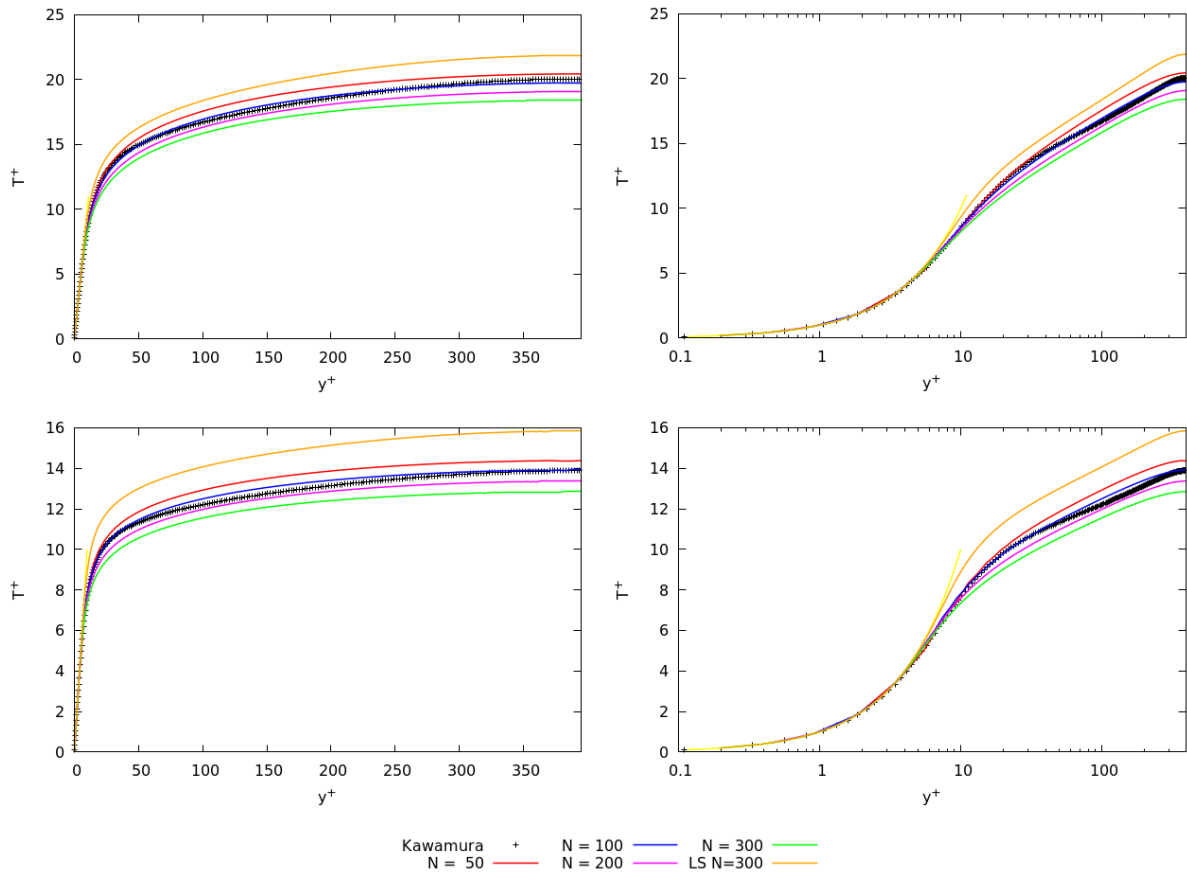
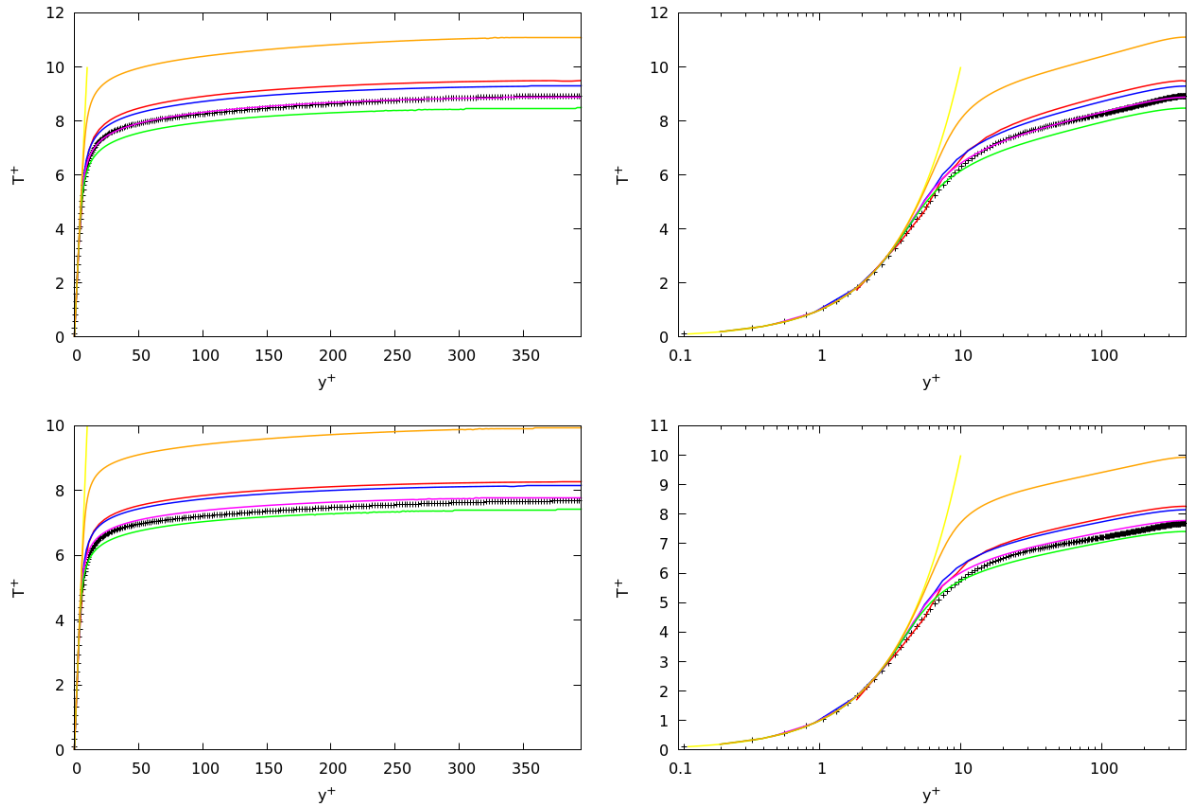


Figure 42: Comparison of temperature field obtained with the four meshes with $k-\omega$ SST momentum turbulence model and Reynolds analogy with $Re_\tau = 395$. From top to bottom the passive scalars are for Prandtl numbers 0.025, 0.71, 1 and 2



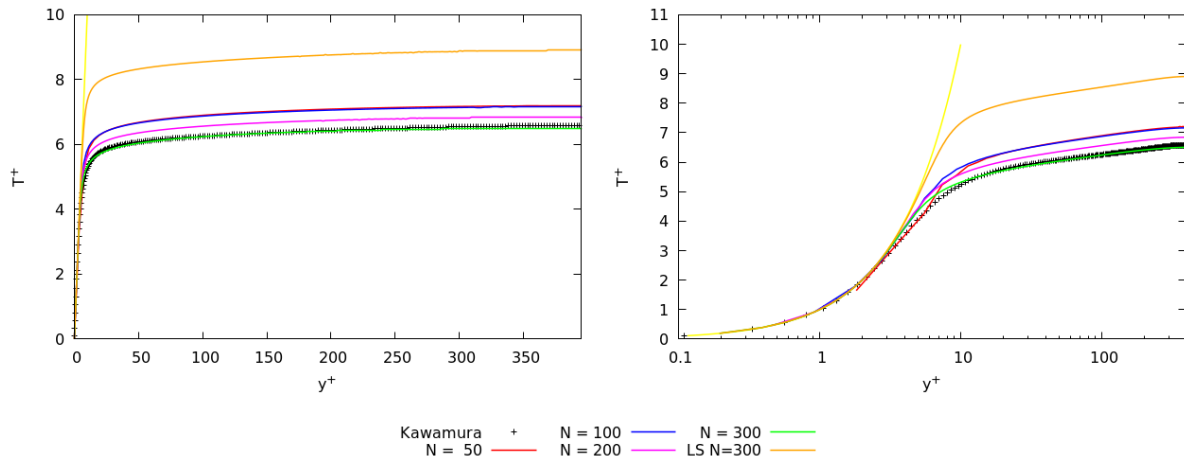


Figure 43: Comparison of temperature field obtained with the four meshes with $k-\omega$ SST momentum turbulence model and Reynolds analogy with $Re_\tau = 395$. From top to bottom the passive scalars are for Prandtl numbers 5, 7 and 10. The barely seen yellow line marks the linear temperature regime.

3.1.2 $Re_\tau = 640$

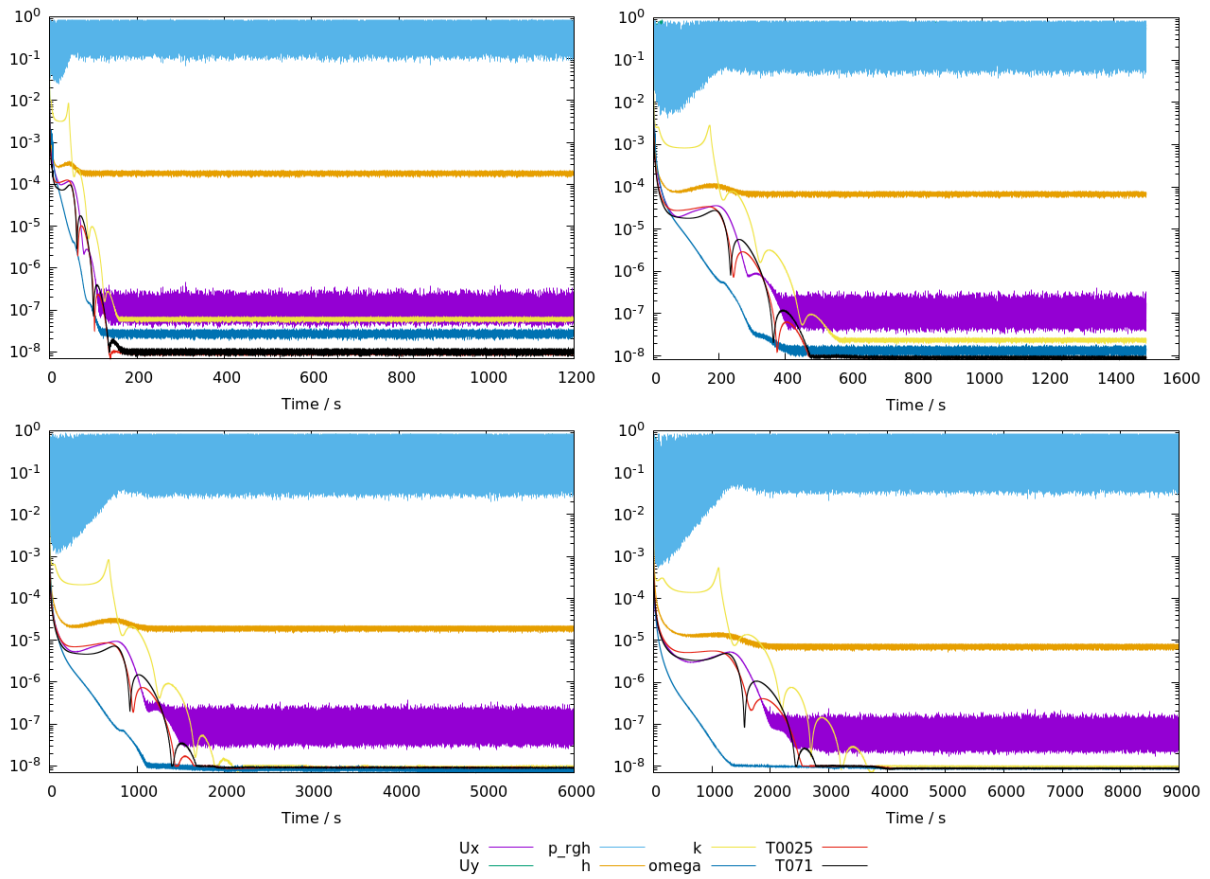


Figure 44: Residuals obtained with meshes $N = 100$ (top left), $N = 200$ (top right), $N = 300$ (bottom left) and $N = 400$ (bottom right) with $k-\omega$ SST model and Reynolds analogy thermal model at $Re_\tau = 640$.

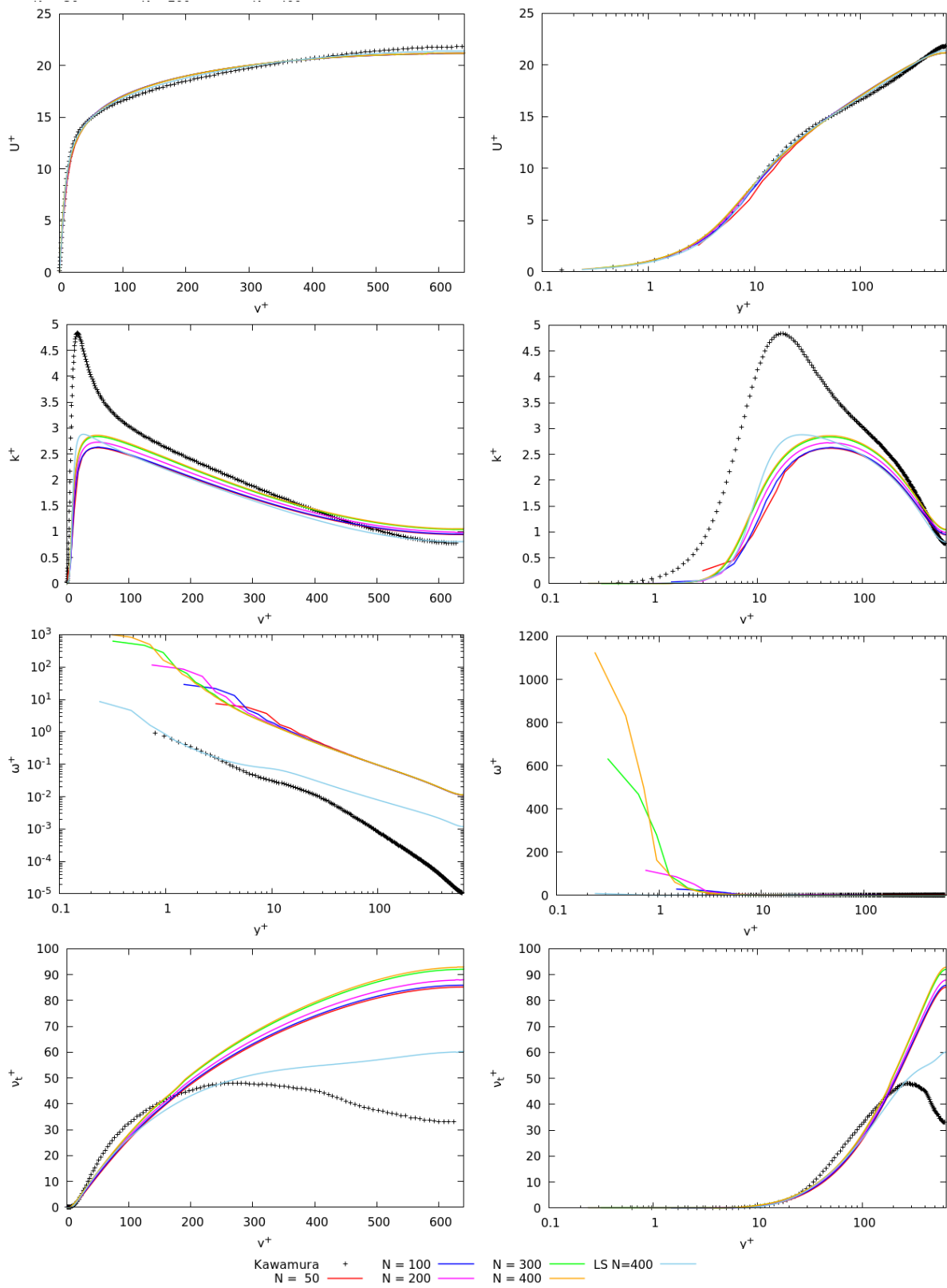


Figure 45: Comparison of momentum and turbulent variables between the four meshes for $k-\omega$ SST model at $Re_\tau = 640$. LS $N = 400$ marks results obtained with Launder-Sharma $k-\varepsilon$ model on the finest mesh.

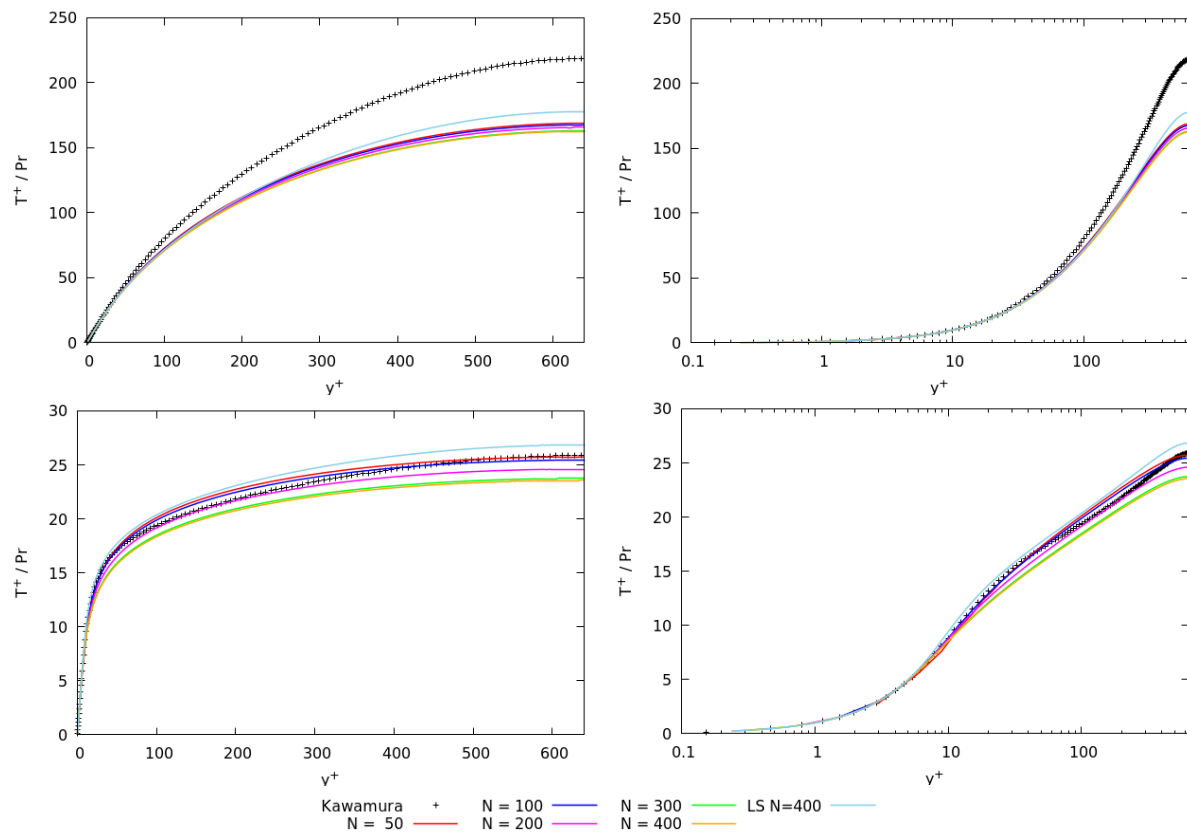


Figure 46: Comparison of temperature field obtained with the four meshes with $k-\omega$ SST momentum turbulence model and Reynolds analogy with $Re_\tau = 640$. From top to bottom the passive scalars are for Prandtl numbers 0.025 and 0.71. The barely seen yellow line marks the linear temperature regime.